ERRATA
for ***Catalan Numbers***

version of 5 June 2018

- p. 1, line 8. Change $d - 1$ to $n - 1$.
- p. 40, item 132. The five examples should be

  12132434   12134234   12314234   12312434   12341234

- p. 51, item 200. The condition on A and B should be that for all i, the ith largest element of A is smaller than the ith largest element of B.
- p. 59, item 17, line 6. Change y to $F(x, t)$ (twice).
- p. 126, line 3 of second triangle. This should be

  1 1 3 7 18

- p. 134, Problem A59. It should be assumed in both parts that $f(x)$ has compact support; otherwise the solution is not unique.
- p. 184, line 10– (omitting footnote). Change *Rendu* to *Rendus*.
- p. 213, column 1, line 4. Change Martin to Michael.

ADDENDA

version of 15 July 2016

B1. (a) [2+] Define integers c_n by

$$C(-x) = \prod_{n\geq 1}(1-x^n)^{c_n}.$$

Show that

$$c_n = \frac{1}{2n}\sum_{d|n}(-1)^{d-1}\mu(n/d)\binom{2d}{d}.$$

(b) [2+] Show that c_n is divisible by n.

(c) [3–] Show that $6c_n$ is divisible by n^2.

B2. [3] Fix $n \geq 2$. Let X be a $(2n-1)$-element set. Let V be the real (any field of characteristic 0 will do) vector space with a basis consisting of all symbols

$$[a_1, \dots, a_i, [b_1, \dots, b_n], a_{i+1}, \dots, a_{n-1}],$$

where $\{a_1, \dots, a_{n-1}, b_1, \dots, b_n\} = X$. Let W be the subspace of V generated by the following elements:

- $[c_1, \dots, c_i, c_{i+1}, \dots, c_{2n-1}] + [c_1, \dots, c_{i+1}, c_i, \dots, c_{2n-1}]$. In other words, the $(2n-1)$-component "bracket" $[c_1, \dots, c_{2n-1}]$ (where each c_i is an element of X with one exception which is a bracket $[b_1, \dots, b_n]$ of elements of X) is antisymmetric in its entries.
- For all $a_1 < \cdots < a_{n-1}$ and $b_1 < \cdots < b_n$ such that $\{a_1, \dots, a_{n-1}, b_1, \dots, b_n\} = X$, the element

$$[a_1, \dots, a_{n-1}, [b_1, \dots, b_n]] - \sum_{i=1}^{n}[b_1, \dots, b_{i-1}, [a_1, \dots, a_{n-1}, b_i], b_{i+1}, \dots, b_n].$$

Show that $\dim V/W = C_n$.

B3. [3–] Let $\boldsymbol{n}$ denote the n-element chain $1 < 2 < \cdots < n$. Show that for $n \geq 3$, C_n is the number of n-element subsets S of the poset $\boldsymbol{n} \times \boldsymbol{n}$ with the following properties: (a) S intersects every maximal chain of $\boldsymbol{n} \times \boldsymbol{n}$ and is minimal with respect to this property, (b) S lies below the *equator*, i.e., if $(i,j) \in S$ then $i + j \leq n+1$, and (c) $(n,1) \in S$.

Solutions

B1. (c) See `http://mathoverflow.net/questions/195339`.

B2. This result was conjectured by Tamar Friedmann (in a more general context) and proved by Phil Hanlon in 2015. Friedmann made the stronger conjecture that the natural $\mathfrak{S}_n$-action on V/W is the irreducible representation indexed by the partition $(2, 2, \ldots, 2, 1)$ of $2n-1$. Hanlon in fact proved this stronger conjecture.

B3. See S. Ahmad and V. Welker, *Order* **33** (2016), 347–358 (Theorem 2.1).