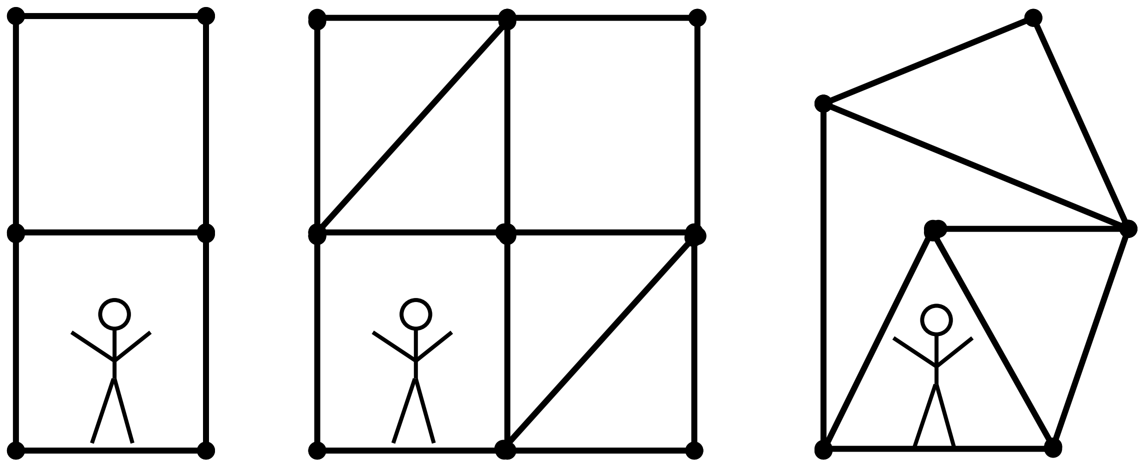
Building
Rigid Trusses Using
Henneberg Construction

Daisy Yuen

Building a House

Want a sturdy frame design

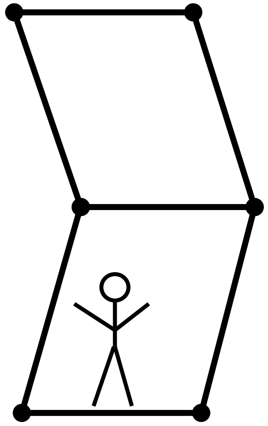


(a) (b) (c)

Three possible designs for the frame of a house

Building a House

Want a sturdy frame design



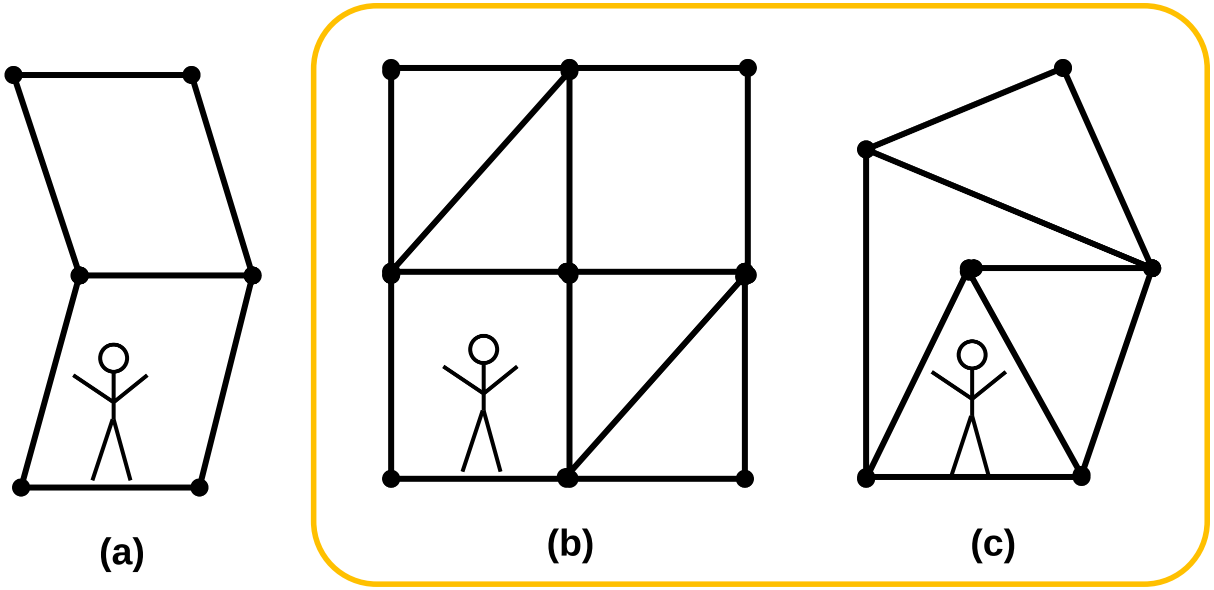
(a)

Shearing!

The shape of a structure is changed without changing the length of any of the structure's members

Building a House

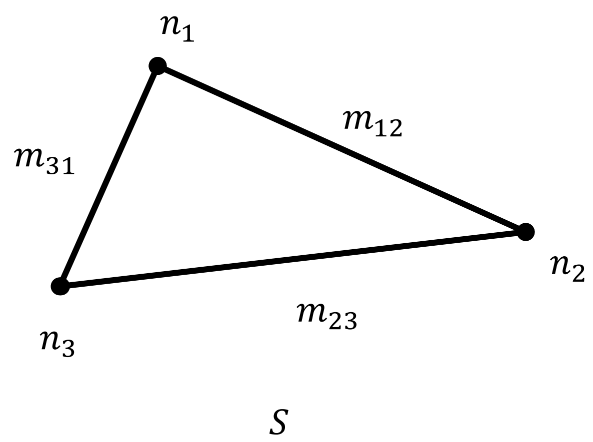
Want a **RIGID** frame – no shearing



Are these rigid?

Use **Laman's Condition**!

Describing Trusses as Graphs



Define:

$$S = \text{graph model for a planar truss},$$
$$M(S) = \text{set of edges of graph } S,$$
$$|M(S)| = \text{number of members of graph } S,$$
$$N(S) = \text{set of nodes of graph } S,$$
$$|N(S)| = \text{number of nodes of graph } S,$$

A **Rigid Truss** is equivalent to a **Stiff Graph**.

Laman's Condition

For any graph σ, we define

$$\mu(\sigma) = 2|N(\sigma)| - |M(\sigma)| - 3$$

Laman's Condition

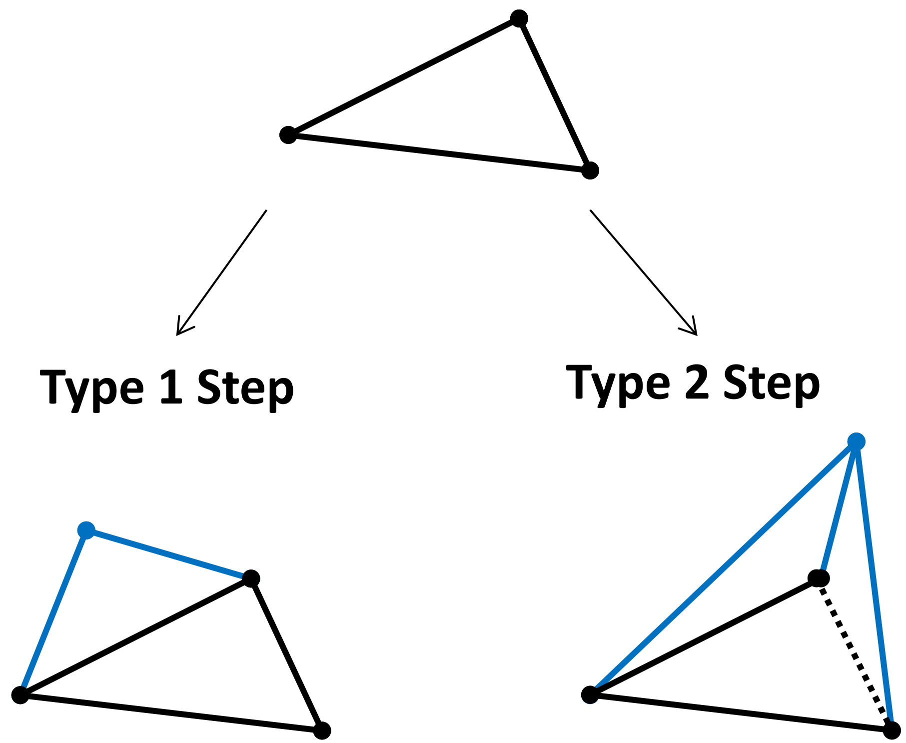
A graph S is stiff if and only if

(a) $\mu(S) = 0$, and

(b) for every non-empty set $X \subseteq M(S)$, $\mu(X) \geq 0$.

Henneberg Construction

Starting with a member, a Laman graph can be built inductively by adding one vertex at a time using one of these two steps:



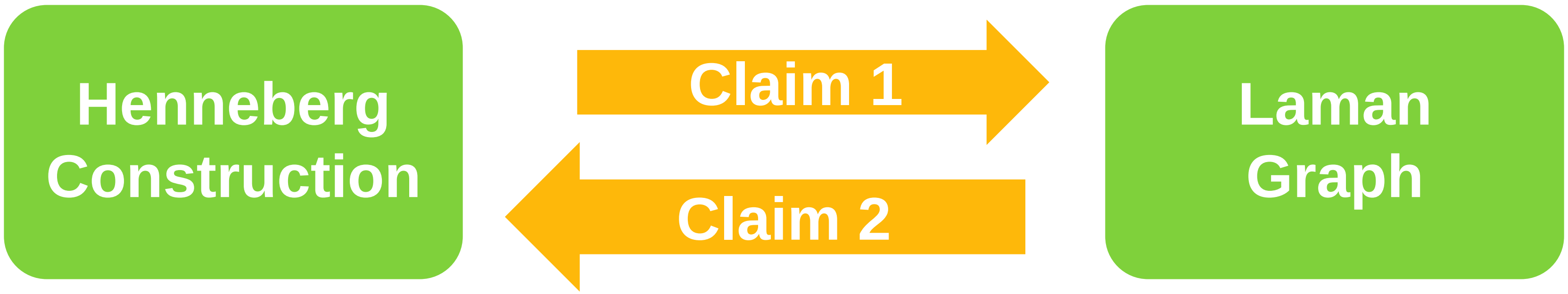
Proving Henneberg Construction

Laman's Condition. A graph S is stiff if and only if

(a) $\mu(S) = 0$, and

(b) for every non-empty set $X \subseteq M(S)$, $\mu(X) \geq 0$.

where $\mu(\sigma) = 2|N(\sigma)| - |M(\sigma)| - 3$



Claim 1. A graph constructed using Henneberg Construction is a Laman Graph

Claim 2. Every Laman graph has a Henneberg Construction