Numerical Analysis on Root finding for Nonlinear Eqn's

(A) Motivation

slide

In thermal dynamics or fluid mechanics, engineers often encounter complicated functions that need to be solved.

For example, to model a wind driven air flow through a room using conservation of mass, I would need to solve an eqn like the following:

after some simplifications:

$$K_1 \sqrt{\frac{2[(P_{out}) - P_{wind,1}) - P_{in}]}{\rho}} - K_2 \sqrt{\frac{2[P_{in} - (P_{out}) - P_{wind,2})}{\rho}} = 0$$

In here, we would need to solve for Pout. But there's no good way to do so by hand. In cases like this, we need to use numerical method, which discretizes continuous functions for computation.

Today, I'm going to talk about some root finding methods that are commonly used and also how fast they converge. numerical analysis.

side board

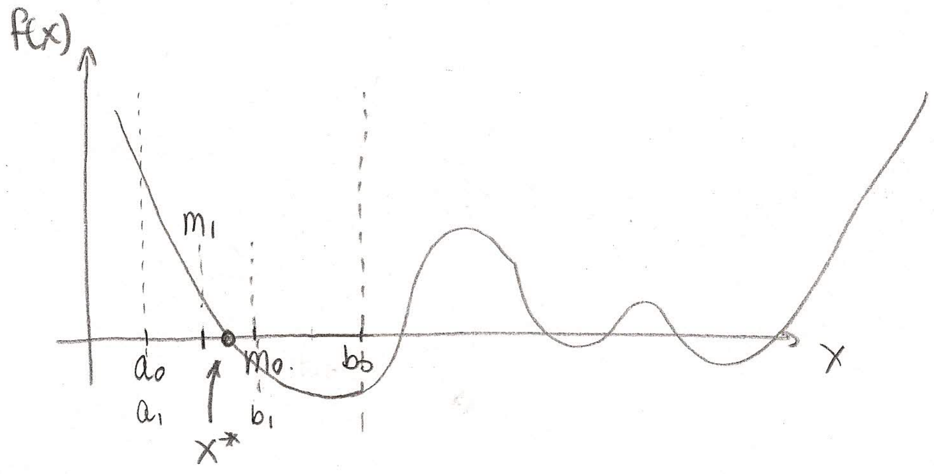
(B) Three common methods.

	PRO'S	CON'S
1. Bisection	can always find a root	slow
2. Newton-Raphson	might fail to find a root	fast
3. Secant.	might fail to find a root	don't need $f'(x)$

① Bisection

Board 1.1.

Given f(x), f'(x) @ any x.



- you want to find x^*.
- you look between interval $[a_0, b_0]$ where $\boxed{a_0 \cdot b_0 < 0}$, meaning one of the values has to be (-) and the other (+). and $\boxed{a_k < x^* < b_k}$
- [In this case $f(a_k) > 0$, $f(b_k) < 0$, where k = # iteration.
- we also want to look for their midpoint

$$m_k = \frac{1}{2}(a_k + b_k)$$

- 3 cases:

  if $f(m_k) < 0$, then

$$a_{k+1} = a_k$$
$$b_{k+1} = m_k.$$

  if $f(m_k) > 0$,

$$a_{k+1} = m_k$$
$$b_{k+1} = b_k$$

  if $f(m_k) = 0$, we've found our solution.

- Repeat until $|f(m_k) - 0| < \varepsilon$.

  ↖ arbitrary # that you set.