

The Lovász Local Lemma and Applications

18.304 – Term paper – first draft

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March 11, 2013

Abstract

A true gem in the probabilistic method is the Lovász Local Lemma, which for a set of bad events A_1, \dots, A_m with bounded dependence degree can guarantee that the event $\bar{A}_1 \wedge \dots \wedge \bar{A}_m$ has positive probability. In this paper we give a proof of the generalized form and show surprising applications of this powerful method to Ramsey theory, hypergraph coloring and the satisfiability problem.

1 Introduction

One of the fundamental techniques in combinatorics is the probabilistic method. To show the existence of a desired object, for example an n -node graph without any $4 \log n$ -size clique or anti-clique, one considers a suitable random experiment to show its existence. . . .

The first application of this technique can be found in . . . , where it was used to show. . . The method was also used in the groundbreaking work of Leighton, Maggs and Rao [LMR94] in the context of packet routing. For a more detailed overview, we refer to the excellent book of Alon and Spencer [AS08].

2 The basic approach

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References

- [AS08] N. Alon and J. H. Spencer. *The probabilistic method*. Wiley-Interscience Series in Discrete Mathematics and Optimization. John Wiley & Sons Inc., Hoboken, NJ, third edition, 2008. With an appendix on the life and work of Paul Erdős.

- [LMR94] F. T. Leighton, B. M. Maggs, and S. B. Rao. Packet routing and job-shop scheduling in $O(\text{congestion} + \text{dilation})$ steps. *Combinatorica*, 14(2):167–186, 1994.