Voting

Xiaoyue Zhang

Preference Ballot

Ordered list of candidates

Assuming no ties

Preference schedule = many preference ballots

Alice's Preferences
1. Apple
2. Banana
3. Peach
4. Pear
5. Kiwi

What is fair?

- Most Satisfied/ Least Dissatisfied
- Last time, we talked about “good matchings” and defined good in terms of “no rogue couples”
- Here, we will try to define a good election result.

What is fair?

- Most Satisfied/ Least Dissatisfied
- Last time, we talked about “good matchings” and defined good in terms of “no rogue couples”
- Here, we will try to define a good election result.

Fairness

- Majority – Guy getting most (>50%) first place votes should be winner
- Condorcet – If A is preferred over B by a majority, then A should be winner
- Monotonicity – If A is winner, A should remain winner if changes to preference schedule that ONLY favor A are made.
- Independence of Irrelevant Alternatives – If A is winner, A should remain winner if a losing candidate drops out and votes are recalculated.

Election Algorithms

- Plurality
- Plurality with Elimination
- Condorcet
- Borda Count

Plurality

# Voters	8	4	3	2
1st	A	B	B	D
2nd	C	D	C	C
3rd	D	C	D	B
4th	B	A	A	A

- Most US elections
- Only care about 1st place votes
- 8 votes for A, so A wins
- Easy to game the algorithm

Plurality with Elimination

At each round, eliminate candidate with FEWEST 1st place votes

	8	4	3	2
1st	A	B	B	D
2nd	C	D	C	C
3rd	D	C	D	B
4th	B	A	A	A

Eliminate C

	8	4	3	2
1st	A	B	B	D
2nd	D	D	D	B
3rd	B	A	A	A

Eliminate D

	8	4	3	2
1st	A	B	B	B
2nd	B	A	A	A

Eliminate A

	8	4	3	2
1st	B	B	B	B

B wins

Abridged version used in France, Georgia, Louisiana

Plurality with Elimination

Violates Monotonicity!

	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

	7	8	10	4
1st	A	C	C	A
2nd	C	A	A	C

	7	8	10	4
1st	C	C	C	C

Moving C up in the rightmost ballot causes C to lose

	7	8	10	4
1st	A	B	C	C
2nd	B	C	A	A
3rd	C	A	B	B

	7	8	10	4
1st	B	B	C	C
2nd	C	C	B	B

	7	8	10	4
1st	B	B	B	B

Also violates IIA: If A dropped out in original election, B would win instead of C

Plurality with Elimination

Violates Monotonicity!

	7	8	10	4
1st	A	B	C	A
2nd	B	C	A	C
3rd	C	A	B	B

	7	8	10	4
1st	A	C	C	A
2nd	C	A	A	C

	7	8	10	4
1st	C	C	C	C

Moving C up in the rightmost ballot causes C to lose

	7	8	10	4
1st	A	B	C	C
2nd	B	C	A	A
3rd	C	A	B	B

	7	8	10	4
1st	B	B	C	C
2nd	C	C	B	B

	7	8	10	4
1st	B	B	B	B

Also violates IIA: If A dropped out in original election, B would win instead of C

Plurality with Elimination

# Voters	8	4	3	2
1st	A	B	B	D
2nd	C	D	C	C
3rd	D	C	D	B
4th	B	A	A	A

- How do voters feel?
  - How many voters prefer C to B?
  - Does not satisfy Condorcet Criterion
- Gaming the System
  - The 8 voters can put candidate D first to avoid getting candidate B.

Condorcet Method

Compare pair of candidates. Winner in pair gets +1 point, loser gets +0, tie gets +0.5 points.

# Voters	8	4	3	2
1st	A	B	B	D
2nd	C	D	C	C
3rd	D	C	D	B
4th	B	A	A	A

	A	B	C	D
A	-	B	C	D
B	B	-	C	D
C	C	C	-	C
D	D	D	C	-
score	0	1	3	2

C wins

Condorcet Method

- Satisfies Condorcet Criterion by construction
- Thus, also satisfies Majority Criterion
- Satisfies Monotonicity:
  - Say A wins. If ONLY A is moved up in a ballot, this could only increase the A's score and no one else's because ONLY A is favored.
- Independence of Irrelevant Alternatives?

Condorcet Method

Does not satisfy IIA.

When C drops out, winner changes from A to B

orig	2	6	4	1	1	4	4
1st	A	B	B	C	C	D	E
2nd	D	A	A	B	D	A	C
3rd	C	C	D	A	A	E	D
4th	B	D	E	D	B	C	B
5th	E	E	C	E	E	B	A

new	2	6	4	1	1	4	4
1st	A	B	B	B	D	D	E
2nd	D	A	A	A	A	A	D
3rd	B	D	D	D	B	E	B
4th	E	E	E	E	E	B	A

orig	A	B	C	D	E
A	-	B	A	A	A
B	B	-	C	tie	B
C	A	C	-	C	E
D	A	tie	C	-	D
E	A	B	E	D	-
score	3	2.5	2	1.5	1

new	A	B	D	E
A	-	B	A	A
B	B	-	tie	B
D	A	tie	-	D
E	A	B	D	-
score	2	2.5	1	1

Borda Count

Assign different points to different places. (Point assignment schemes can control results: most satisfied/least dissatisfied)

Our scheme: ith place gets n-i points

# Voters	8	4	3	2
1st	A	B	B	D
2nd	C	D	C	C
3rd	D	C	D	B
4th	B	A	A	A

A: 8*3 + 4*0 + 3*0 + 2*0 pts = 24 pts

B: 8*0 + 4*3 + 3*3 + 2*1 pts = 23 pts

C: 8*2 + 4*1 + 3*2 + 2*2 pts = 30 pts

D: 8*1 + 4*2 + 3*1 + 2*3 pts = 25 pts

C wins!

Borda Count

Violates Majority and Condorcet criteria!

# Voters	6	2	3
1st	A	B	C
2nd	B	C	D
3rd	C	D	B
4th	D	A	A

A: 6*3 + 2*0 + 3*0 = 18 pts

B: 6*2 + 2*3 + 3*1 = 21 pts

C: 6*1 + 2*2 + 3*3 = 19 pts

D: 6*0 + 2*1 + 3*2 = 8 pts

B wins, but A is Majority and Condorcet candidate.

Borda Count

- In reality, violates fairness criterion relatively infrequently.
- Used in Icelandic Parlimentary Elections, and main method in Slovenia, Kiribati, Nauru
- Most popular method for ranking sports teams and colleges

Different Methods and Fairness

Method → Fairness ↓	Plurality	Plu. w Elim	Condorcet	Borda Count
Majority	yes	yes	yes	no
Condorcet	no	no	yes	no
Monotonicity	yes	no	yes	What do you think?
IIA	no	no	no	What do you think?

Arrow's Impossibility Theorem

Fair voting system (w/ at least 3 candidates):

- Can handle any preference schedule
- Deterministic
- Consensus – if all voters prefer A to B, society prefers A to B
- Impartiality – all candidates treated equally
- IIA
- No dictators – if single voter prefers A to B while all others prefer B to A, society must prefer B to A.

DOES NOT EXIST!

Arrow's Impossibility Theorem

- Def quorum: group of voters such that all voters in quorum prefer A to B, and all voters not in quorum prefers B to A, and society prefers A to B
- If S and T are quorums, S intersect T is also a quorum
- V – {v} must be a quorum, or else v is a dictator
- Thus, {} is a quorum, which violates consensus

Acknowledgments

http://www.mscf.uky.edu/~lee/ma111fa11/slides01.pdf

http://en.wikipedia.org/wiki/Condorcet_method#Basic_procedure

http://en.wikipedia.org/wiki/Arrow's_impossibility_theorem

http://www.math.ucla.edu/~tao/arrow.pdf