Counting with Rotational Symmetry

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Abstract

We consider the problem of counting a set of structures with rotational symmetry. By applying Burnside's Lemma, we count the number of orbits in the set rather than the number of elements. From this problem we derive intuition for the proof of Burnside's Lemma. Finally we briefly discuss applications to the field of chemistry.

1 Introduction

You are a chemical engineer and you have just synthesized a new compound, X. You hypothesize that your compound, when combined with benzene, will cure cancer. More specifically, you want to replace one or more of the carbons in a benzene ring with X, inject the resulting compound into mice, and watch a miracle happen.

Since benzene has six carbons, each of which you may choose to replace with X, you calculate that you need to conduct $2^6 = 64$ experiments.

That night, you're grabbing a beer with your best friend, who is a math major. You tell her about your plan. Your friend informs you that you need only _____ experiments! Why is this so?

2 Formalization

1. The stabilizer of $s = 101010$ is $Stab_s = \{g_0, g_2, g_4\}$
2. The set of elements of S fixed by $g = g_2$ is $Fix_{g_2} = \{111111, 101010, 000000, 010101\}$
3. The orbit of $s = 101010$ is $Orb_s = \{101010, 010101\}$

3 Solution

1. $Fix_{g_0} = S$
2. $Fix_{g_1} = \{111111, 000000\}$

3. $Fix_{g_2} = \{111111, 101010, 000000, 010101)\}$
4. $Fix_{g_3} = \{000000, 101101, 010010, 100100, 110110, 111111, 001001, 011011\}$
5. $Fix_{g_4} = \{111111, 101010, 000000, 010101)\}$
6. $Fix_{g_5} = \{111111, 000000\}$

4 Intuition

The multiorbit of 000000 is $\{000000, 000000, 000000, 000000, 000000, 000000\}$.