

examples

We have a bag we want to fill w/ n fruits

Fruits: Apples, Bananas, Oranges, Pears.

- Apples even
- Bananas Multiples of 5
- Oranges At most 4
- At most 1 Pear

$$n=4$$

fruit	# of fruits
Apples	4 2 0 0
Bananas	0 0 0 0
Oranges	0 1 2 4 3
Pears	0 1 0 0 1

Ways of filling basket w/ 4 fruits.

$$A(x) = 1 + x^2 + x^4 + x^6 + \dots = \frac{1}{1-x^2}$$

$$B(x) = 1 + x^5 + x^{10} + x^{15} + \dots = \frac{1}{1-x^5}$$

$$O(x) = \underbrace{1 + x + x^2 + x^3 + x^4}_{\text{finite sequence}} = \frac{1-x^5}{1-x} \quad \text{introduce trick}$$

$$P(x) = 1 + x \quad (\text{we can select 0 Pear or 1 Pear})$$

$$\begin{aligned}
 A(x) B(x) O(x) P(x) &= \frac{1}{1-x^2} \frac{1}{1-x^5} \frac{1-x^5}{1-x} (1+x) \\
 &= \frac{(1+x)}{1-x^2} \frac{1}{(1-x)} = \frac{(1+x)}{(1-x)^2} \\
 &= \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \dots
 \end{aligned}$$

Verify this 0 fruits 1 way 1 fruit 2 ways 2 or