

## Fibonacci Generating Function

0 1 1 2 3 5 ...

$f_0 f_1 f_2 f_3 \dots$

$$F(x) = f_0 x^0 + f_1 x + f_2 x^2 + f_3 x^3 + f_4 x^4 + \dots$$

$$F(x) = x + x^2 + 2x^3 + 3x^4 + \dots$$

$$xF(x) = x^2 + x^3 + 2x^4 + \dots$$

$$x^2 F(x) = x^3 + x^4 + \dots$$

$$F(x) - xF(x) - x^2 F(x) = x \text{ (everything else cancels because of the recurrence } f_n = f_{n-1} + f_{n-2} \text{ or } f_n - f_{n-1} - f_{n-2} = 0)$$

$$F(x) = x / (1 - x - x^2)$$