Homework 8 Solutions

Problems

1. **(a) Express $\binom{7}{4} + \binom{7}{5}$ as a single binomial coefficient.**

   (b) Express $\binom{9}{4} + \binom{9}{3} + \binom{10}{3}$ as a single binomial coefficient.

   Thinking about how the binomial coefficients fit into Pascal's triangle, and knowing that each row of Pascal's triangle is obtained from the row immediately before it by adding pairs of elements together we see that the answers are $\boxed{\binom{8}{5}}$ and

$$\binom{9}{4} + \binom{9}{3} + \binom{10}{3} = \binom{10}{4} + \binom{10}{3} = \boxed{\binom{11}{4}}$$

2. **Consider the first 7 rows of Pascal's triangle (i.e. Pascal's triangle up to and including the row 1 6 15 20 15 6 1).**

   (a) If you choose a random entry from this part of Pascal's triangle, what is the probability that it's a 1?

   (b) If you choose a random entry from this part of Pascal's triangle, waht is the probability that it is even?

   The first 7 rows of Pascals' triangle are

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

The probability of choosing a 1 is $\boxed{\dfrac{13}{28}}$, while the probability of choosing an even number is $\boxed{\dfrac{9}{28}}$.

It wasn't part of the problem, but notice the pattern of even numbers in Pascal's triangle:

1

1 1

1 **2** 1

1 3 3 1

1 **4** **6** **4** 1

1 5 **10** **10** 5 1

1 **6** 15 **20** 15 **6** 1

1 7 21 35 35 21 7 1

1 **8** **28** **56** **70** **56** **28** **8** 1

1 9 **36** **84** **126** **126** **84** **36** 9 1

1 **10** 45 **120** **210** **252** **210** **120** 45 **10** 1

1 11 55 165 **330** **462** **462** **330** 165 55 11 1

1 **12** **66** **220** 495 **792** **924** **792** 495 **220** **66** **12** 1

1 13 **78** **286** 715 1287 **1716** **1716** 1287 715 **286** **78** 13 1

1 **14** 91 **364** 1001 **2002** 3003 **3432** 3003 **2002** 1001 **364** 91 **14** 1

1 15 105 453 1365 3003 5005 6435 6435 5005 3003 1365 453 105 15 1