Homework 6 Solutions

Problems

1. **You roll two standard 6-sided dice. What is the probability that their sum is at least 8? What is the probability that their difference is at least 2?**

   There are 6^2 total possible throws. The throws whose combined score is above 8 are

   $$(2,6),(3,5),(3,6),(4,4),(4,5),(4,6),(5,3),(5,4),(5,5),(5,6),(6,2),(6,3),(6,4),(6,5),(6,6)$$

   which is 15 possible throws. Hence the probability of throwing a sum of at least 8 is $\boxed{\frac{15}{36} = \frac{5}{12}}$.

   There are 6 throws with difference zero. Given any dice roll of 1-5, we can get a throw with difference one by increasing that result by 1. The set of all such throws is obtained by these, and the swapping of which die comes first (these only include the ones where the first die roll is lower). So the total number of throws with difference one is $5 \cdot 2 = 10$, and the total with difference at least 2 is $36 - 6 - 10 = 20$. So the probability is $\boxed{\frac{20}{36} = \frac{5}{9}}$.

2. **You've come up with the following game to play with a friend. Both players will flip three coins. Whoever gets more heads wins. Of course, it is possible that both players get the same number of heads. In that case, a player chosen beforehand wins the game. In order to make up for this advantage, the tie-winning player only gets one dollar for winning (regardless of whether there was actually a tie or not), while the tie-losing player gets two. Would you rather lose ties, or win them?**

   Let us first figure out what the probabilities are for the results of one person's coin flips. There are a total of $2 \cdot 2 \cdot 2 = 8$ possible sequences, of which $\binom{3}{0} = 1$ have no heads, $\binom{3}{1} = 3$ have one head, $\binom{3}{2} = 3$ have two heads, and $\binom{3}{3} = 1$ have three heads. Therefore, the probability that both players get no heads is $\frac{1}{8} \cdot \frac{1}{8} = \frac{1}{64}$, that both get one head is $\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$, that both get two heads is $\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$, and that both get three heads is $\frac{1}{8} \cdot \frac{1}{8} = \frac{1}{64}$. Thus the probability of a tie is $\frac{1}{64} + \frac{9}{64} + \frac{9}{64} + \frac{1}{64} = \frac{20}{64}$, and the probability that someone wins is $1 - \frac{20}{64} = \frac{44}{64}$. Of those, the tie winning player will win half: $\frac{1}{2} \cdot \frac{44}{64} = \frac{22}{64}$. Thus, the total probability of winning for the tie winning player is $\frac{20}{64} + \frac{22}{64} = \frac{42}{64}$, which is slightly less than half of the tie-losing player's chance of winning: $\frac{22}{64}$. Since the tie-losing player receives twice as much money when they win, you would rather $\boxed{\text{lose ties}}$.

3. **Suppose that you flip a coin 10 times. Which of the following sequences of heads and tails is more likely?**

   (a) THHTTHTHTH

   (b) TTTTTTTTTH

   Every sequence of ten coin flips is equally likely. These two sequences have $\boxed{\text{the same}}$ probability of occuring.

4. **Suppose that you flip a coin 10 times. Which is more likely:**

   (a) You get 5 heads and 5 tails.

   (b) You get 2 heads and 8 tails.

Since the denominator is the same (2^{10}), we need only compare the numerators. The number of sequences of flips that have five heads and five tails is the same as the number of ways to choose which of the ten spots should be heads, which is

$$\begin{aligned} \binom{10}{5} &= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= 2 \cdot 3 \cdot 7 \cdot 6 \\ &= 252. \end{aligned}$$

Similarly, the number of ways to get 2 heads and 8 tails is $\binom{10}{2} = 45$. So $\boxed{\text{5 heads and 5 tails}}$ is more likely.