

# Homework 5 Solutions

## Problems

1. **How many distinct ways are there to rearrange the letters in “MISSISSIPPI”? How about “WOOLLOOMOOLOO”**

In MISSISSIPPI there are 1 M, 2 P's, 4 I's, and 4 S's. To rearrange these letters involves having 11 slots and choosing one of the slots for the M, 2 for the P's, 4 for the I's, and 4 for the S's. There are

$\left( \begin{array}{c} 11 \\ 4, 4, 2, 1 \end{array} \right)$  ways of doing this. Similarly for WOOLLOOMOOLOO we

see there are  $\left( \begin{array}{c} 13 \\ 8, 3, 1, 1 \end{array} \right)$  ways of rearranging the letters.

2. (a) **To pass the time you of course play card games. As you prepare for a game of war, you quickly scan through the deck and notice that no two spades are next to each other. How many arrangements of the deck have this property? (NB: a standard deck has four suits of thirteen cards each)**
- (b) **Later you begin to tire of card games and together construct a rudimentary house out of palm trees and grass. In the house there is one room that would sleep 5 people, one that sleeps 4, two doubles, and a single. In this house will live your 14 “children” which are really just coconuts that you have decorated. In how many ways can you assign your children to the rooms?**

There are 39 non-spade cards in the deck. The number of arrangements of the cards with no two spades adjacent is the same as the number of ways of inserting spades into the gaps between these non-spades. There are 40 gaps (allowing a gap on the top and bottom of the deck), so there are a total of  $\left( \begin{array}{c} 40 \\ 13 \end{array} \right)$  arrangements. We also need to multiply by the reorderings of the

spades and the nonspades, yielding  $\left( \begin{array}{c} 40 \\ 13 \end{array} \right) \cdot 13! \cdot 39!$  total arrangements.

The second part is asking how ways are there of dividing up a group of 13 children into 5 groups of 5,3,2,2, and 1. There are  $\left( \begin{array}{c} 13 \\ 5, 3, 2, 2, 1 \end{array} \right)$  ways of doing this.