Homework 3 Solutions

Problems

1. **The American Students' Phonetic Association is trying to increase membership by pretending to be a fraternity. As part of this subterfuge it wants to pick a three-letter fraternity name but with names drawn from the phonetic alphabet which has 107 letters. How many names can be formed?**

   How many names can be formed if each of the three letters has to be different?

   What if we only disallow names where all three letters are the same (like "ððð") but all other combinations are okay - how many possible names are there then?

   For the first part we use the multiplication principle: $\boxed{107^3}$.

   For the second part we use the multiplication principle without repetition: $\boxed{107 \cdot 106 \cdot 105}$.

   The final part is most easily tackled using the subtraction principle. There are 107 possible names with all three letters being the same, hence the number of allowable names is $\boxed{107^3 - 107}$.

2. **How many three-digit numbers are there?**

   How many three-digit numbers are divisible by 3? How many three-digit numbers are divisible by 7?

   How many three-digit numbers are neither divisible by 3 nor by 7? (*Hint:* A number is divisible by *both* 3 and 7 exactly when it is divisible by 21.)

   We can tackle the first part by counting - the number of numbers between 100 and 999 is $\boxed{999 - 100 + 1 = 900}$.

   We do the second part by counting as well. The number of numbers divisible by 3 between 100 and 999 is the number of numbers divisible by 3 between 102 and 999. This (dividing by 3) is the same as the number of numbers between 34 and 333 which is $\boxed{333 - 34 + 1 = 300}$. Similarly, the number of numbers divisible by 7 is $\boxed{142 - 15 + 1 = 128}$.

   For the final part we use a *multiple subtraction*. The number of numbers divisible by both 3 and 7 in the range is the number of numbers divisible by 21 between 105 and 987 which is (dividing by 21) 47-5+1 = 43. Hence the number of numbers neither divisible by 3 nor 7 is $\boxed{900 - 300 - 128 + 43 = 515}$.

3. **The most popular club in Harvard is, of course, the Diet Coke and Mentos Pyrotechics Club. Membership, as you know, is very exclusive. The Club always consists of 100 members: 10 drawn from the freshman, 20 from the sophomores, 30 from the juniors, and 40 from the seniors. Graduate students are vigorously barred.**

   The club wishes to elect officers: the President, Vice-President, Mentos Master, and Diet Coke Panjandrum. How many ways are there to do this if no-one is allowed to hold more than one position?

   How many ways are there if the Presidential position has to be filled by a senior?

   How many ways are there if at least one of the four positions has to be filled by a senior?

   For the first part we use the multiplication principle without repetition: $\boxed{100 \cdot 99 \cdot 98 \cdot 97}$.

   For the second part we use the multiplication principle again. We imagine that the presidential position is the first to be filled, then leaving a pool of 99 for vice-president and so on. $\boxed{40 \cdot 99 \cdot 98 \cdot 97}$

We use the subtraction principle for the final part. There are $60 \cdot 59 \cdot 58 \cdot 57$ ways of filling the positions using *no* seniors, hence there are $\boxed{100 \cdot 99 \cdot 98 \cdot 97 - 60 \cdot 59 \cdot 58 \cdot 57}$ ways of filling the positions using *at least one* senior.