

Homework 28 Solutions

Problems

1. **Estimate how many primes there are less than a billion (10^9). Your answer will be considered correct if it is within 20%.**

The actual number of primes less than a billion is 50847534. The rough estimate of $\frac{10^9}{\log(10^9)} = \boxed{48254942}$ is easily within 20%. The better estimate using a logarithmic integral is given by 50849234, which is quite good.

The lower bound on the correct answer is 40678028, while the upper bound is 61017036.

2. **A number b between 1 and $n-1$ is called a *witness* for the fact that n is composite if $b^{n-1} \not\equiv 1 \pmod{n}$. (remember: if n were prime, Fermat's little theorem would say that $b^{n-1} \equiv 1 \pmod{n}$). So n has to be composite if it has any witness).**

- (a) **Show that 3 fails to be a witness for 91 to be composite.**

We compute (using doubling) that $3^{90} \equiv 1 \pmod{91}$, hence 3 is not a witness to the fact that 91 is composite.

- (b) **Show that 2 is a witness for 255 to be composite.**

We compute (using doubling) that $2^{254} \equiv 64 \pmod{255}$ hence 2 is a witness to the fact that 255 is composite.