Homework 25 Solutions

Problems

1. **(a) Compute** $3^{917} \pmod{140}$**.**

   (b) Compute $4^{1125} \pmod{105}$**.**

   $\phi(140) = (1/2)(4/5)(6/7)140 = 48$. So $3^{48} = 1 \pmod{140}$. So $3^{917} = 3^5 = 243 = \boxed{103} \pmod{140}$.

   $\phi(105) = (2/3)(4/5)(6/7)105 = 48$. So $4^{48} = 1 \pmod{105}$. So $4^1125 = 4^{21} \pmod{105}$ and this can be figured out by doubling to be $\boxed{64} \pmod{105}$.

2. **The method we have used for computing roots** $\pmod{n}$ **can be applied to only part (c) of the following three problems. Explain why it fails for the first two problems, and solve the third.**

   (a) The 4th root of $4 \pmod{77}$**;**

   (b) The 7th root of $7 \pmod{77}$**;**

   (c) The 13th root of $13 \pmod{77}$**.**

   4 and $\phi(77) = 60$ are not relatively prime so we can't take 4th roots.

   7 and 77 are not relatively prime so we can't find roots of 7.

   $1/13 = 37 \pmod{60}$. So $13^{1/13} = 13^{37} \pmod{77}$ and this can be figured out by doubling to be $\boxed{62} \pmod{77}$.

3. **(a) Compute the 11th root of** $2 \pmod{105}$**.**

   (b) Compute the 5th root of $4 \pmod{39}$**.**

   First observe that $105 = 3 \times 5 \times 7$ so that $\phi(105) = 48$. This is a problem modulo 105, so this means that we work modulo 48 in the exponent. We wish to compute $2^{1/11} \pmod{105}$. First we figure out (using Euclid's algorithm backwards) that $1/11 = 35 \pmod{48}$. So the answer will be $2^{35} \pmod{105}$, which we can compute using doubling to be $\boxed{53} \pmod{105}$.

   Note that $39 = 3 \cdot 13$ so $\phi(39) = 2 \cdot 12 = 24$ and we work modulo 24 in the exponent. We determine that $1/5 = 5 \pmod{24}$ so $4^{1/5} \equiv 4^5 \equiv \boxed{10} \pmod{39}$.