

# Homework 19 Solutions

## Problems

### 1. Find all the reciprocals of non-zero values mod 17.

$x$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$1/x$	1	9	6	13	7	3	5	15	2	12	14	10	4	11	8	16

First note that  $16 \equiv -1$  so  $16^{-1} \equiv 16$ .

Then note that  $1 \equiv 18 \equiv 35 \equiv 52 \pmod{17}$ .

$9 \times 2 \equiv 18 \equiv 1 \pmod{17}$  so 9 and 2 are inverses.

$3 \times 6 \equiv 18 \equiv 1 \pmod{17}$  so 3 and 6 are inverses.

$7 \times 5 \equiv 35 \equiv 1 \pmod{17}$  so 7 and 5 are inverses.

$4 \times 13 \equiv 52 \equiv 1 \pmod{17}$  so 4 and 13 are inverses.

Also,  $(-2)^{-1} = -2^{-1}$  so  $-2 \equiv 15$  and  $-9 \equiv 8$  are inverses.

Similarly,  $(-3)^{-1} = -3^{-1}$  so  $-3 \equiv 14$  and  $-6 \equiv 11$  are inverses.

The remaining pair, 10 and 12 must be inverses (though we can check:  $10 \times 12 \equiv 120 \equiv 1 \pmod{17}$ ).

We could have also done some of these using  $(ab)^{-1} \equiv a^{-1}b^{-1}$  or using the Euclidean algorithm.

### 2. Compute the following powers mod 13.

(a)  $2^0, 2^1, 2^2, \dots, 2^{15}, 2^{16}$ .

(b)  $3^0, 3^1, 3^2, \dots, 3^{15}, 3^{16}$ .

First note that  $2^0 \equiv 1$ ,  $2^1 \equiv 2$ ,  $2^2 \equiv 4$ ,  $2^3 \equiv 8$ , and  $2^4 \equiv 16 \equiv 3 \pmod{13}$ .

Then  $2^5 \equiv 2 \times 3 \equiv 6$  and  $2^6 \equiv 2 \times 6 \equiv 12 \equiv -1$ , which means that the next 6 values will be negatives of the first:

$2^7 \equiv -2 \equiv 11$ ,  $2^8 \equiv -4 \equiv 9$ ,  $2^9 \equiv -8 \equiv 5$ ,  $2^{10} \equiv -3 \equiv 10$ ,  $2^{11} \equiv -6 \equiv 7$ , and  $2^{12} \equiv -(-1) \equiv 1$ .

Thereafter they repeat, i.e.,  $2^{13} \equiv 2^1 \equiv 2$ ,  $2^{14} \equiv 2^2 \equiv 4$ ,  $2^{15} \equiv 2^3 \equiv 8$ , and  $2^{16} \equiv 2^4 \equiv 3$ .

$3^0 \equiv 1$ ,  $3^1 \equiv 3$ ,  $3^2 \equiv 9$ ,  $3^3 \equiv 27 \equiv 1$ , and they repeat from here.