Homework 17 Solutions

Problems

1. **Do the following computations in the given modulus.**

   (a) $6 - 4 \pmod 7$**.**

   (b) $80 + 21 \pmod{101}$**.**

   (c) $3 - 12 \pmod{15}$**.**

   (d) $456 \times 450 \pmod{457}$**.**

   $\boxed{2}$, $\boxed{0}$, $\boxed{6}$, $-1 \times -7 \equiv \boxed{7}$

2. **Do the following computations in the given modulus.**

   (a) $457 \times 458 \pmod{459}$.

   (b) $13 \times 44 \pmod 5$.

   (c) $13 \times 44 \pmod{15}$.

   In all three cases, we have $(-2) \times (-1) \equiv \boxed{2}$ in the given modulus.

3. **This problem concerns a divisibility rule for 4, ie a way of telling if a number is divisible by 4 easily.**

   (a) Show why the following divisibility rule for 4 works:
   Add the last digit to twice the second to last digit. If the sum is a multiple of four, then the number is a multiple of 4.
   For example, 16 is a multiple of 4 since $6 + 2 \cdot 1 = 8$ **is a multiple of 4.**
   [Hint: We can write 538 as $5 \cdot 10^2 + 3 \cdot 10 + 8$**. What does considering this expression modulo 4 tell us?]**

   (b) Is 2736 divisible by 4? Why or why not?

   (c) Is 293847102938470192834701928374 divisible by 4? Why or why not?

   We write out a general number $a_n a_{n-1} \cdots a_2 a_1 a_0$ as

   $$a_n \cdot 10^n + a_{n-1} \cdot 10^{n-1} + \cdots + a_2 \cdot 100 + a_1 \cdot 10 + a_0.$$

   Now note that 100 is divisible by 4, and so is 1000, etc. Rephrasing this using modular arithmetic, we have $100 \equiv 0 \pmod 4$, $1000 \equiv 0 \pmod 4$, etc. Moreover, $10 \equiv 2 \pmod 4$. Thus

   $$\begin{aligned} a_n \cdot 10^n + a_{n-1} \cdot 10^{n-1} + \cdots + a_2 \cdot 100 + a_1 \cdot 10 + a_0 &\equiv a_n \cdot 0 + a_{n-1} \cdot 0 + \cdots + a_2 \cdot 0 + a_1 \cdot 10 + a_0 \\ &\equiv a_1 \cdot 10 + a_0 \\ &\equiv a_1 \cdot 2 + a_0 \end{aligned}$$

   But if two numbers are congruent modulo 4, then one is divisible by 4 if and only if the other one is. So our original number is a multiple of 4 if and only if the sum of its last digit plus twice the second to last digit is a multiple of 4.

   We now apply the rule.

   (a) 2736 is a multiple of 4, since $3 \cdot 2 + 6 = 12$ is.

   (b) 293847102938470192834701928374 is not a multiple of 4 since $7 \cdot 2 + 4 = 18$ is not.