Homework 10 Solutions

Problems

1. **Use the Euclidean algorithm to find the greatest common divisor of 154 and 374.**

$$\begin{aligned} 374 &= 2 \times 154 && +66 \\ 154 &= 2 \times 66 && +22 \\ 66 &= 3 \times \boxed{22} \end{aligned}$$

Therefore gcd(154, 374) = 22.

2. **Use the Euclidean algorithm to find the greatest common divisor of 2377 and 1284.**

$$\begin{aligned} 2377 &= 1 \times 1284 && +1093 \\ 1284 &= 1 \times 1093 && +191 \\ 1093 &= 5 \times 191 && +138 \\ 191 &= 1 \times 138 && +53 \\ 138 &= 2 \times 53 && +32 \\ 53 &= 1 \times 32 && +21 \\ 32 &= 1 \times 21 && +11 \\ 21 &= 1 \times 11 && +10 \\ 11 &= 1 \times 10 && +1 \\ 10 &= 10 \times \boxed{1} \end{aligned}$$

Therefore gcd(2377, 1284) = 1.

3. (a) **Use the Euclidean algorithm to find the greatest common divisor of 13 and 31.**

$$\begin{aligned} 31 &= 2 \times 13 && +5 \\ 13 &= 2 \times 5 && +3 \\ 5 &= 1 \times 3 && +2 \\ 3 &= 1 \times 2 && +1 \\ 2 &= 2 \times \boxed{1} \end{aligned}$$

Therefore gcd(13,31)=1.

(b) **Express 1 as a combination of 13 and 31.**

$$\begin{aligned} 1 &= 3 - 2 \\ &= 3 - (5 - 3) && = 2 \times 3 - 5 \\ &= 2 \times (13 - 2 \times 5) - 5 && = 2 \times 13 - 5 \times 5 \\ &= 2 \times 13 - 5 \times (31 - 2 \times 13) && = \boxed{12 \times 13 - 5 \times 31} \end{aligned}$$

(c) **Express 2 as a combination of 13 and 31.**

$$\begin{aligned} 2 &= 2 \times (12 \times 13 - 5 \times 31) \\ &= \boxed{24 \times 13 - 10 \times 31} \end{aligned}$$

4. (a) **Use the Euclidean algorithm to find the greatest common divisor of 57 and 87.**

$$\begin{aligned} 87 &= 1 \times 57 + 30 \\ 57 &= 1 \times 30 + 27 \\ 30 &= 1 \times 27 + 3 \\ 27 &= 9 \times \boxed{3} \end{aligned}$$

Therefore $\gcd(57, 87) = 3$.

(b) **Express 3 as a combination of 57 and 87.**

$$\begin{aligned} 3 &= 30 - 27 \\ &= 30 - (57 - 1 \times 30) \qquad\qquad = 2 \times 30 - 57 \\ &= 2 \times (87 - 57) - 57 = \boxed{2 \times 87 - 3 \times 57} \end{aligned}$$

(c) **Can 1 be expressed as a combination of 57 and 87? Why or why not?**

$\boxed{\text{No,}}$ because any whole number that is a combination of 57 and 87 is a multiple of their GCD, 3.