

Quantitative Reasoning 28:

The Magic of Numbers

Homework **28**

Assigned on Friday December 5th

Due at 12 noon **Wednesday** December 10th

Please submit problem sets at the end of the relevant lecture, or leave in the box labeled QR28 outside the Math Department's main office, on the third floor of the Science Center (Room 325).

Reading:

Gross-Harris, Chapter 23

Problems:

Please explain your reasoning and show your work.

1. Estimate how many primes there are less than a billion (10^9). Your answer will be considered correct if it is within 20%.
2. A number b between 1 and $n - 1$ is called a *witness* for the fact that n is composite if $b^{n-1} \not\equiv 1 \pmod{n}$. (remember: if n were prime, Fermat's little theorem would say that $b^{n-1} \equiv 1 \pmod{n}$. So n has to be composite if it has any witness).
 - (a) Show that 3 fails to be a witness for 91 to be composite.
 - (b) Show that 2 is a witness for 255 to be composite.