Practice Questions for 2nd Midterm

Please do not get discouraged; this is longer and harder than the real midterm will be.

1. (a) Use the Euclidean Algorithm to find the greatest common divisor of 44 and 17.

   (b) Find whole numbers x and y so that $44x + 17y = 1$ with $x > 10$.

   (c) Find whole numbers x and y so that $44x + 17y = 1$ with $y > 10$.

2. For each of the following four parts say whether there are whole numbers x and y satisfying the equation. If an equation has a solution, write down a possible choice of x and y.

   (a) $69x + 123y = 2$.

   (b) $47x + 21y = 2$.

   (c) $47x - 21y = 6$.

   (d) $49x + 21y = 6$.

3. (a) What is the largest prime number dividing the binomial coefficient $\binom{12}{4}$?

   (b) How many divisors does $\binom{12}{4}$ have?

   (c) How many of the divisors of $\binom{12}{4}$ are divisible by 3?

4. Let $m = 1100$ and $n = 2^2 \times 3^3 \times 5^5$.

   (a) Compute $\gcd(m, n)$.

   (b) Compute $\mathrm{lcm}(m, n)$.

   (c) How many whole numbers divide m but not n?

   (d) How many whole numbers divide n but not m?

5. Do the following calculations. As always, when working mod n, leave your answer in the range $0, 1, \ldots, n - 1$.

   (a) $7 \cdot 9 \pmod{36}$.

   (b) $8 - 21 \pmod{31}$.

   (c) $68 \cdot 69 \cdot 71 \pmod{72}$.

   (d) $108! \pmod{83}$.

   (e) $60^{59} \pmod{61}$.

   (f) $1/2 \pmod{17}$.

   (g) $1/11 \pmod{43}$.

6. (a) Compute $21^{4600} \pmod{47}$.

   (b) Compute $21^{4601} \pmod{47}$.

   (c) Compute $21^{4599} \pmod{47}$. (Hint: your work on 2(b) will help).

7. (a) Compute $87^{51} \pmod{47}$.

(b) Compute $94^{46} \pmod{47}$.

8. (a) Find an x between 0 and 19 such that $x^2 \equiv 5 \bmod 19$.
   (b) What does Fermat's theorem say about powers of x?
   (c) Compute $5^9 \bmod 19$.

9. (a) Use the Euclidean Algorithm to find the reciprocal of $40 \bmod 93$. Check your work by verifying that your answer is in fact a solution of $40x \equiv 1 \bmod 93$.
   (b) Using your answer to the first part, find the reciprocals mod 93 of 4 and 89. (Hint: $4 + 89 = 93$.)

10. The goal of this problem is to find reciprocals mod 23 for all the nonzero numbers mod 23. Record your answers in the table below.

x	1	2	3	4	5	6	7	8	9	10	11
$1/x$	1										

x	12	13	14	15	16	17	18	19	20	21	22
$1/x$											

   (a) What is $\frac{1}{22} \bmod 23$?
   (b) Use the fact that $2^{11} \equiv 2048 \equiv 1 \bmod 23$ to find the reciprocals of 2, 4, 8, and 16.
   (c) Fill in the rest of the table.

11. Please make the requested computations modulo 11 putting your answers in the range

$$\{0, 1, 2, \ldots, 10\}.$$

   (a) Find 3^{12} modulo 11.
   (b) Find $2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \pmod{11}$.
   (c) Does a solution to the equation

$$5^{10} y \equiv 6^{61} \pmod{11}$$

   exist? If it does, please find it.

12. In analogy with the divisibility rule for 11, can you come up with a divisibility rule for 1001? (Hint: it will only be useful for really large numbers)

    NB: Since $1001 = 7 \cdot 11 \cdot 13$, this gives a "simultaneous" divisibility test for those three primes, which is useful for large numbers. Also note that we won't ask you about divisibility rules on the test, but thinking through this problem can help you understand the concepts from this unit.