MATH 220 Final Exam Sample 1

1) Find the inverse function $f^{-1}(x)$ of $f(x) = \frac{x+1}{2x+1}$

2) Eliminate the parameter to find a Cartesian Equation of the curve $x = 2 - 3t, \quad y = 4 - t^2$. Identify the Cartesian Curve.

3) Given the function $f(x) = |x|x^{\frac{1}{3}}$, use the Definition of the Derivative to decide whether $f(x)$ is differentiable at $x = 0$, and if so, find $f'(0)$.

4) Find the derivatives of the following functions. It is not necessary to simplify your answers.

a) $f(x) = 4\sin(5x) - \frac{1}{\sqrt[3]{x^2}} + \ln(3x)$

b) $g(x) = \frac{e^{2x}}{1+x^2}$

c) $y = x^3 \tan^{-1}(4x)$

d)$y = (x+1)^x$

e) $h(t) = \ln[(3t+2)^5(t-4)]$

f) $f(x) = \int_0^x \frac{\sin t dt}{t^3+2}$

5) Find an equation of the tangent line to the curve

$$x^2 + 4xy + y^3 = 13 \qquad \text{at the point} \quad (2, 1).$$

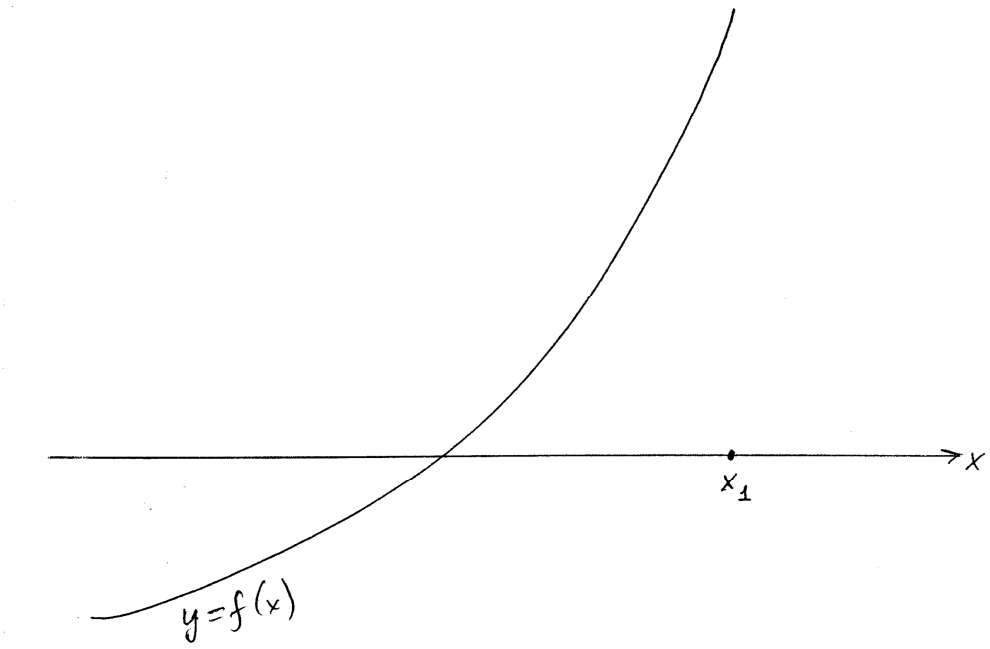
6) Consider the function $f(x) = xe^{-x}$.
a) Find all horizontal and vertical asymptotes, if any.
b) Determine on what intervals $f(x)$ is increasing or decreasing.
c) Determine all local maximum and minimum values.
d) Determine on what intervals $f(x)$ is concave up and concave down, and determine all inflection points, if any.
e) Sketch the graph of $f(x)$.

7) The length of a rectangle is increasing at a rate of 8 cm/sec and its width is increasing ata rate of 3 cm/sec. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

8) Find the absolute maximum and the absolute minimum values of $f(x) = x^4 - 2x^2 + 3$ on the closed interval $[-2, 3]$.

9) A rectangle has two its vertices on the $x-$axis and the other two on the graph of the parabola $y = 16 - x^2$. Of all possible rectangles, find the dimensions of the one with maximum area.

10) We use Newton's Method to find the approximate value of the solution of the equation $f(x) = 0$. The graph of the function f and the position of x_1 is shown. Draw on the same picture the positions of $x_2,\ x_3$, and x_4.



11) Find the linear approximation for the function $f(x) = \sqrt[9]{x}$ at $a = 1$ and use it to approximate $\sqrt[9]{1.1}$.

12. Evaluate the following limits. Show your work!

a) $\lim_{x\to 2-} \frac{|x-2|}{x^2-4}$

b) $\lim_{x\to\infty} \frac{3x^2-x-2}{5x^2+4x+1}$

c) $\lim_{x\to 0} \frac{\sin(6x)}{\ln(x+1)}$

d) $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$

e) $\lim_{x\to 0+} (1+3x)^{1/x}$

13. Given $f''(t) = 2e^t + 3\sin t,\ f(0) = 0,\ f(\pi) = 0$. Find $f(t)$.

14. Use the Midpoint Rule with $N = 4$ to approximate $\int_1^5 \frac{1}{\sqrt{x^3+1}}\, dx$. DO NOT SIMPLIFY YOUR ANSWER!

15. Evaluate the following definite integrals.

a) $\int_1^9 \frac{3x-1}{\sqrt{x}}\, dx.$

b) $\int_0^2 t\sqrt{4+t^2}\, dt.$

c) $\int_{-2}^2 \sqrt{4-x^2}\, dx.$ (HINT: Use Geometry!)

16. Evaluate the following indefinite integrals.

a) $\int \frac{\ln x}{x}\, dx.$

b) $\int x \ln x\, dx.$

c) $\int \sin^3 x\, dx.$