

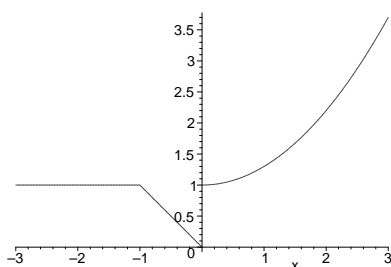
Practice Final for Calculus I (M0220)

1. Derive $\frac{d}{dx} \frac{1}{x} = \frac{-1}{x^2}$ from the definition of the derivative as the limit of the difference quotient.
2. Find each limit (or report that it does not exist) by inspecting the given graph of $y(x) = f(x)$

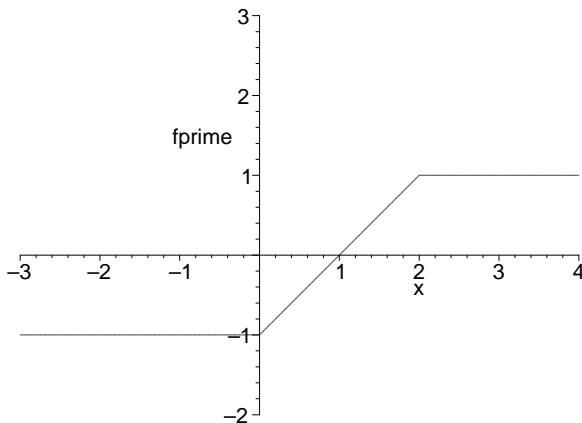
$$\lim_{x \rightarrow -1} f(x) =$$

$$\lim_{x \rightarrow 0^+} f(x) =$$

$$\lim_{x \rightarrow 0} f(x) =$$



3. You are given the graph of $f(x)$. Draw on the same axes the graph of the antiderivative passing through $(0,0)$.



4. A box with a square base and open top must have a volume of $V = 32,000 \text{ cm}^3$. Find the dimensions of the box that minimizes the amount of material used.
5. Two cars start moving from the same point. One travels south at 60 mph and the other travels west at 25 mph. At what rate is the distance between the cars increasing two hours later?

6. Find the derivative of each of the given functions $y = f(x)$ (do not simplify)

(a) $\sqrt{x^2 - 3 \cos 2x - 5}$

(b) $\frac{7x-5}{3x^3+5x^2-2x+1}$

(c) $4x^5 e^{-.2x}$

(d) $\ln 1 + \cos^2(3x - 1)$

(e) $y = f(x)$ defined implicitly by $xy + 3x^2y^3$

7. Find the slope of the line tangent to the curve $x = 4t^2$ $y = \cos t$ when $t = \frac{\pi}{2}$

8. Find each antiderivative and use it to evaluate each definite integral.

(a)

$$\int_0^{\pi} \cos \theta d\theta$$

(b)

$$\int_0^1 \frac{1}{1+x^2}$$

(c)

$$\int_1^2 e^{3x} dx$$

(d)

$$\int_1^8 \sqrt[3]{x}(x-1)$$

9. Use calculus to sketch the graph of $y = x^4 - 4x^3$ and indicate where it is concave up and concave down, decreasing and increasing. Use $y'(x)$ to find and verify critical points. Use $y''(x)$ to find and verify inflection points.

10. Find each limit exactly (i.e write π not 3.14 and $\ln 7$ not 1.94591) and show the derivation.

(a)

$$\lim_{x \rightarrow 0} \frac{6^x - 2^x}{x}$$

(b)

$$\lim_{x \rightarrow 2} \frac{6^x - 2^x}{x}$$

(c)

$$\lim_{x \rightarrow \infty} \frac{x^2 - 3}{x^2 + x + 5}$$

(d)

$$\lim_{x \rightarrow 0} \frac{5}{x - 3}$$

(e)

$$\lim_{x \rightarrow 0} \frac{2x - \cos 3x}{e^{3x} - 1}$$

11. Find the $\sqrt[3]{11}$ by Newton's Method on your calculator and record the iterations. Hint: use $f(x) = x^3 - 11$

(a) $x_1 =$

(b) $x_2 =$

(c) $x_3 =$

12. Compute the left Riemann Sum L_4 of $y = x^3$ on $[0, 2]$ with 4 equal intervals.