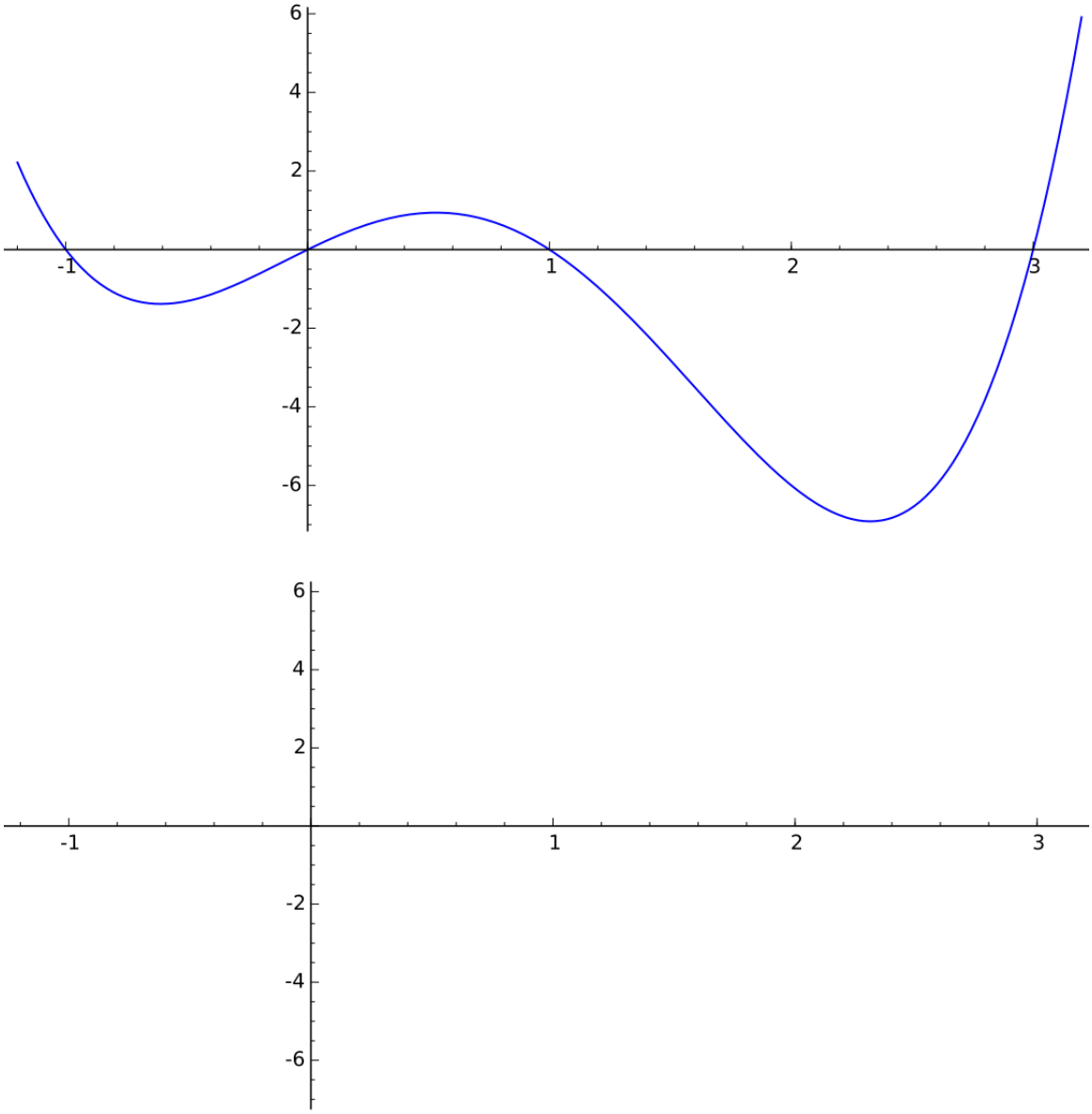
Math 220 - Practice Exam 1

1. Give the limit if it exists
   (a) $\lim_{x\to 2-} \frac{|x^2-2x|}{x-2}$
   (b) $\lim_{x\to 2+} \frac{|x^2-2x|}{x-2}$
   (c) $\lim_{x\to 2} \frac{\sqrt{x+2}-2}{x-2}$
2. Determine the derivatives of the following functions:
   (a) $f(x) = x^4 - 2\sqrt{x} + 1 - \frac{2}{x^3} + \pi x^\pi$,
   (b) $f(x) = (x^2 - 4x + 5)^6(2x^3 - 1)^3$,
   (c) $g(x) = \frac{3}{(x-\sin(x))^2}$,
   (d) $f(x) = \frac{\sin(x)\tan(3x)}{1+\cos^2(x)}$,
   (e) $h(x) = \sin^3(x^3)$.
3. Evaluate the limit
$$\lim_{h\to 0} \frac{\tan(\pi/4 + h) - 1}{h}.$$
4. If $f(x) = x^3 - 2x + 1$,
   (a) What is the differential of $f(x)$ at $x = 2$?
   (b) $f(2) = 5$. Use the differential to approximate $f(1.9)$.
5. Use implicit differentiation to determine the equation of the tangent line to the curve
$$x^3y^2 - x^2y^3 + 2x^2 - y^2 = 1$$
   at the point $(1, 1)$.
6. A $10m$ ladder leans against a wall. As the base of the ladder is pulled away at a rate of $0.6m/s$, the ladder slides down the wall. At what rate is the height of the top of the ladder changing when the base of the ladder is $6m$ from the wall? (Give an exact numerical answer)
7. Determine where the function $f(x) = x\sqrt{8 - x^2}$ has a horizontal tangent line.

8. Sketch the graph of the derivative of the given function $f(x)$:



9. The chart below gives the values of $f(x)$ and $g(x)$ and their derivatives at $x = 2$, $x = 3$ and $x = 4$.

	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	4	-2	3	9
3	0	29	-1	4
4	3	2	7	11

Use it to find the following derivatives,

(a) If $h(x) = f(g(x))$, determine $h'(4)$.
(b) If $k(x) = g(f(x))$, determine $k'(4)$.