Math 220 - Fall 2010 Exam 1 Solutions

1. (a) $x^2 - 5x + 6 = (x-2)(x-3)$ so $\lim_{x\to 2} \frac{x^2-5x+6}{x-2} = 2 - 3 = -1$.

   (b) For x slightly larger than 2, the numerator is negative and the denominator positive. So $\lim_{x\to 2^+} \frac{x^2-5x+5}{x-2} = -\infty$.

   (c) For $x < 2$, $x^2 - 5x + 6 > 0$, so $\lim_{x\to 2^-} \frac{|x^2-5x+6|}{x-2} = \lim_{x\to 2^-} \frac{(x-2)(x-3)}{x-2} = -1$.

   (d) The degree of the numerator is less than the degree of the denominator, so $\lim_{x\to\infty} \frac{3x^2-7x+1}{x^3-1} = 0$.

2. (b) $f'(x) = 3(4x-7)^2(4)(7x^2+4)^4 + 4(4x-7)^3(7x^2+4)^3(14x)$.

   (c) $f'(x) = \frac{3(x+5\tan(3x)) - 3x(1+15\sec^2(3x))}{(x+5\tan(3x))^2}$.

   (d) $f'(x) = 12\sin^2(4x+1)\cos(4x+1)$.

4. $A = LW$. Differentiating with respect to time, we have

$$A' = L'W + W'L = (2)(8) + (-3)(10) = -14.$$

   The units are square feet per second.

5. Implicitly differentiating with respect to x,

$$3(x+2y)^2(1+2y') + 3(2x+y)^2(2+y') + 2y + 2xy' = 0.$$

   Substituting for x and y and solving for y',

$$\begin{aligned} 3(-1+2)^2(1+2y') + 3(-2+1)^2(2+y') + 2 - 2y' &= 0 \\ 3 + 6y' + 6 + 3y' + 2 - 2y' &= 0 \\ 7y' &= -11 \\ y' &= \frac{-11}{7} \end{aligned}$$

   Thus the tangent line is

$$y - 1 = \frac{-11}{7}(x+1).$$

6. Let $f(x) = \sqrt{x}$. Since $x = 99.6$ is close to $a = 100$, where f is easy to evaluate, we use linear approximation there. We have $f'(a) = \frac{1}{2\sqrt{a}} = \frac{1}{20}$. Then

$$\begin{aligned} \sqrt{99.6} = f(x) &\approx f(a) + f'(a)(x-a) \\ &= 10 + \frac{1}{20}(99.6 - 100) \\ &= 10 - 0.02 \\ &= 9.98. \end{aligned}$$

7. The derivative is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}.$$

   $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ is also acceptable.