PROBLEM SET 1, FOR 18.102, FALL 2007
DUE THURSDAY 13 SEPT, AT 2:30 IN CLASS (2-102).

All homework should either be submitted to me by email (it does not have to be in TeX, scanned written pages are fine) or on paper of reasonable quality, so it goes through the scanner! If you need decent paper I am happy to supply some.

Unfortunately I am working from the 2nd edition and many of you have the third. The problems are Chapter 1, problems 3, 7, 8, 9, 11 and here they are written out:-

(1) (No.3) Prove that a subspace of a vector space is a vector space itself. [The point of course is to write this out carefully but succinctly].

(2) (No.7) Show that any vector of $\mathbb{R}^3$ is a linear combination of (the) vectors $(1,0,0)$, $(1,1,0)$ and $(1,1,1)$. [Once again, try to keep it brief and as clear as you can]

(3) (No.8) Prove that every quadruple of (i.e. set of four) vectors in $\mathbb{R}^3$ is linearly dependent. [Meaning show there is a non-trivial linear relation between them.]

(4) (No.9) Prove that the functions $f_n(x) = x^n$, $n \in \{0, 1, \dots\}$ are linearly independent.

(5) (No.11) Prove that each of the spaces $\mathcal{C}(\mathbb{R}^n)$, $\mathcal{C}^k(\mathbb{R}^n)$, $\mathcal{C}^\infty(\mathbb{R}^n)$ is infinite dimensional. [Try the case n=1 first, the other cases follow from this; also think about which of the three cases is the hardest and how it is related to the others, before you start!]