LECTURE 1

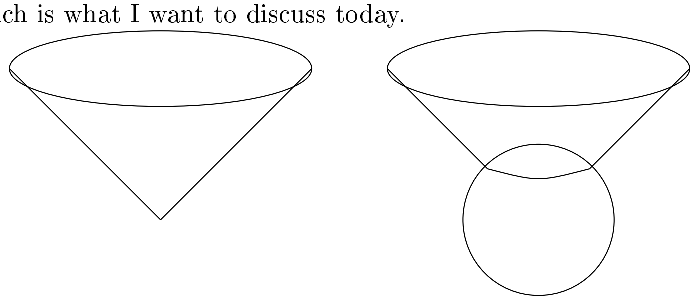
Why blow things up? – And the definition

Today I want to define the basic process of 'blowing up' a manifold around a submanifold. What I will describe is the real version of a procedure that is well known to algebraic geometers in the complex setting. In fact there are several variants, the main one is *radial blow up* which is what I will talk about almost exclusively. There is also the closely related *projective blow up* which is very similar, except one trades off the non-introduction of boundaries for a loss of orientability. I will indicate at some point why there are some reasons to prefer the radial procedure but in essence they are equivalent. There is also the notion of *parabolic blow up* which is similar but different – I will indicate what this is about but will probably not have time to go through it in any detail.

So, the basic question is:- Why blow up at all? If one is working in a genuinely smooth and uniform analytic setting there is not much reason to blow anything up. However, there are three closely related circumstances in which blow up can be very helpful. These correspond to trying to 'resolve'

(1) A singular function, e.g. $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.
(2) A singular space, e.g. $C = \{t^2 = x^2 + y^2,\ t \geq 0\}$
(3) Degenerate vector fields, e.g. the span of $z_j\partial_{z_j}$ $j = 1, 2, 3$ on $\mathbb{R}^3$.

In all three cases these can be resolved by the introduction of polar coordinates – which is what I want to discuss today.



If one is constrained to work on a singular space – for instance the (one-sided) cone in Euclidean space C pictured above then one has a problem doing anything much right at the singular point. One can choose to work in small neighbourhoods away from, in some appropriate sense uniformly up to, the singular point but it is difficult to work directly around the singular point. A basic question for instance is: What is the space of smooth functions on C? In fact it is fair to say that there is no single answer to this but that the most obvious one is not very good. Namely one could say that a function on C is smooth if it is the restriction to C of a smooth function on $\mathbb{R}^3$. Then however, the usual properties of coordinate systems and Taylor series and so on fail, or get much more complicated.

In real blow up, the idea is simply to work in polar coordinates around the singular point. That is, we lift everything up to a manifold with boundary by using the polar map

$$\beta : [0, \infty) \times C_1 \ni (r, \theta) \longrightarrow r\theta \in C. \tag{1.1}$$

Here C_1 is the circle in $\mathbb{S}^2$ given by the intersection of C with the sphere of radius 1 in $\mathbb{R}^2$:

$$C_1 = C \cap \{t^2 + x^2 + y^2 = 1\} = \{(t, x, y) = (\frac{1}{\sqrt{2}}, \frac{\theta}{\sqrt{2}}),\ \theta \in \mathbb{R}^2,\ |\theta| = 1\}. \tag{1.2}$$

It is a (normalized) cross-section of the cone.

Now, $r > 0$ on the left in (1.1) is mapped diffeomorphically onto the smooth part of the cone; this is clear enough since it is immediate for the restriction to $r = 1$ and scaling in r on the left corresponds to radial scaling on the right. Thus the cone is 'blown up' to a manifold with boundary, where the whole boundary is mapped to the conic point under the 'blow down map' β. In fact we are really blowing up the ambient space, $\mathbb{R}^3$, and seeing what happens to the singular subset C. Notice most of all that the blow-down map β is itself smooth, it is the inverse of β which is singular – to the extent that it is undefined near $r = 0$.

So, what we are doing here is blowing up the origin in $\mathbb{R}^3$ and 'lifting' the previously singular subset to a smooth manifold with boundary. This is a procedure that works with great generality, when applied with sufficient diligence and care – as in Hironaka's remarkable result which asserts that by *appropriate* iteration of the complex version of this construction one can render any projective algebraic variety smooth.

1. Polar coordinates

The basic example of blow up then is to introduce polar coordinates around the origin in $\mathbb{R}^n$. Thus the model blow-down map in codimension n is

$$\beta : [0, \infty) \times \mathbb{S}^{n-1} \ni (r, \omega) \longmapsto r\omega \in \mathbb{R}^n. \tag{1.3}$$

Here $\mathbb{S}^{n-1}$ is the unit sphere in $\mathbb{R}^n$. This map is smooth! It is a diffeomorphism from $(0, \infty) \times \mathbb{S}^{n-1}$ onto the complement of the centre 0, i.e. onto $\mathbb{R}^n \setminus \{0\}$. On the other hand, the whole of the 'front face' $r = 0$ is mapped into the centre $\{0\}$. What other interesting features does this map have? Of course, it is not 1-1 so does not have an inverse but it is surjective.

For smooth maps there is a general notion of 'related vector fields'. Namely if $f : X \longrightarrow Y$ is a smooth map between manifolds (or open sets in Euclidean space if you prefer) then the differential $F_* : T_xX \longrightarrow T_{f(x)}Y$ is well-defined at each point. A vector field V on X and a vector field W on Y are f-related if $f_*(V_x) = W_{f(x)}$ for all $x \in X$.

LEMMA 1. *For every smooth vector field on $\mathbb{R}^n$ which vanishes at the origin, there is a unique smooth vector field on $[0, \infty) \times \mathbb{S}^{n-1}$ with is β-related to it.*

PROOF. Computing on the sphere is a bit tricky so I will not try to do it here! In fact this result is really a consequence of the homogeneity of β so let me give a full proof which involves little work. First, what is a smooth vector field on $\mathbb{R}^n$?

It is a smooth section of the tangent bundle, and hence a combination of the basic vector fields $\partial/\partial z_j$, $j = 1, \ldots, n$ with smooth coefficients

$$W = \sum_j a_j(z) \frac{\partial}{\partial z_j}. \tag{1.4}$$

So, what does it mean for W to vanish at the origin? It means that each of the coefficients $a_j(z)$ must vanish at $z = 0$. By Taylor's theorem this means exactly that there are smooth functions a_{ij}, $i, j = 1, \ldots, n$ such that $a_j(z) = \sum_i a_{ij}(z) z_i$. Thus if W vanishes at the origin it can be written as a linear combination with smooth coefficients of the n^2 vector fields $z_i \partial_{z_j}$:

$$W = \sum_{ij} a_{ij}(z) z_i \frac{\partial}{\partial z_j}. \tag{1.5}$$

Now, it is a general fact that if a is a smooth function on the image space of $f : X \longrightarrow Y$ and V and W are f-related then $(f^* a)V$ and aW are f-related. This just comes from the fact that f_* is the transpose of f^* which is the pull-back on differential 1-forms.

Thus we only need to show that each of the vector fields $W_{ij} = z_i \partial_{z_j}$ is β-related to some smooth vector field V_{ij} on $[0, \infty) \times \mathbb{S}^{n-1}$. Such a vector field is of the form

$$V_{ij} = b_{ij}(r, \theta)\partial_r + T_{ij}(r) \tag{1.6}$$

where $b_{ij} \in \mathcal{C}^\infty([0, \infty) \times \mathbb{S}^{n-1})$ and the $T_{ij}(r)$ are smooth vector fields on the sphere depending smoothly on r as a parameter. Now β is a diffeomorphism in $r > 0$ so V_{ij} exists and is unique in $r > 0$. This is where the homogeneity completes the proof. Under the scaling diffeomorphism $r \longmapsto \tau r$, $\tau > 0$ the vector field V_{ij} changes to

$$V_{ij} = b_{ij}(\tau r, \theta)\tau^{-1}\partial_r + T_{ij}(\tau r) \tag{1.7}$$

but on the image this is the scaling $z \longmapsto \tau z$ under which W_{ij} is invariant. Thus from the uniqueness of the V_{ij} in $r > 0$ we see that

$$b_{ij}(\tau r, \theta) = \tau b_{ij}(r, \theta),\ T_{ij}(\tau r) = T_{ij}(r)\ \forall\ \tau, r > 0. \tag{1.8}$$

Thus the $T_{ij}(r) = T_{ij}(1)$ are independent of r and $b_{ij}(r, \theta) = r b_{ij}(\theta)$ is linear in r and hence

$$V_{ij} = b_{ij}(\theta) r \partial_r + T_{ij},\ b_{ij} \in \mathcal{C}^\infty(\mathbb{S}^{n-1}) \tag{1.9}$$

with the T_{ij} smooth vector fields on the sphere. Since $r\partial_r$ is certainly a smooth vector field, the V_{ij} are smooth down to $r = 0$ as claimed. □

In fact we conclude a little more from this proof than just the lifting. Namely we can say that the smooth vector fields on $\mathbb{R}^n$ which vanish at the origin lift to unique smooth vector fields on $[0, \infty) \times \mathbb{S}^{n-1}$ and that the lifted vector fields span, over $\mathcal{C}^\infty([0, \infty) \times \mathbb{S}^{n-1})$ *all* the smooth vector fields on $[0, \infty) \times \mathbb{S}^{n-1}$ which are tangent to the boundary. Why is this so? Well a smooth vector field on $[0, \infty) \times \mathbb{S}^{n-1}$ is of the form (1.6) as already noted. To be tangent to $r = 0$ the coefficient of ∂_r must vanish at $r = 0$ and hence it must be of the form

$$b_{ij}(r, \theta) r \partial_r + T_{ij}(r). \tag{1.10}$$

As is well-known, or can be proved directly, $z \cdot \partial_z = \sum_i z_i \partial_{z_i}$ lifts to $r\partial_r$ – since these are the generators of the respective radial actions. Thus the first term in (1.10) is in the span of the lift over $\mathcal{C}^\infty([0,\infty) \times \mathbb{S}^{n-1})$. It also follows from this that all the constant (in r) vector fields on the sphere, T_{ij} are in the span of the lift. Now, these must span the smooth vector fields on the sphere, since β is a diffeomorphism for $r > 0$ and this finishes the proof.

2. Change of coordinates

So, this blow-up and smooth blow-down map (1.3) have nice properties which can be stated invariantly – the lifting of vector fields vanishing at the centre and for instance that the inverse image of the centre is the boundary with a smooth defining function r. What about coordinate invariance? Really it is coordinate-invariance which makes blow up important and separates it from 'just introducing polar coordinates' (although that is precisely what we are doing).

LEMMA 2. *If U_1 and U_2 are open neighourhoods of $0 \in \mathbb{R}^n$ and $F : U_1 \longrightarrow U_2$ is a diffeomorphism such that $F(0) = 0$ then there is a diffeomorphism*

$$\tilde{F} : \tilde{U}_1 = \{(r,\theta) \in [0,\infty) \times \mathbb{S}^{n-1}; r\theta \in U_1\} \longrightarrow \tilde{U}_2 = \{(r,\theta) \in [0,\infty) \times \mathbb{S}^{n-1}; r\theta \in U_2\} \tag{1.11}$$

giving a commutative diagram

$$\begin{tikzcd} \tilde{U}_1 \arrow[r, "\tilde{F}"] \arrow[d, "\beta"'] & \tilde{U}_2 \arrow[d, "\beta"] \\ U_1 \arrow[r, "F"] & U_2. \end{tikzcd} \tag{1.12}$$

Clearly $\tilde{F}$ is unique if it exists, since it is determined by continuity from $r > 0$.

PROOF. If F is an orthogonal transformation then $\tilde{O}$ is just the restriction of O to $\mathbb{S}^{n-1}$ acting trivially on r. In particular this means that we can replace F by OF if necessary to arrange that $L = F_*(0) \in \mathrm{GL}(n,\mathbb{R})$ is orientation-preserving and so is connected to the identity by a smooth curve L_t, $t \in [0,1]$ so $L_0 = \mathrm{Id}$, $L_1 = F_*(0)$. The vector field, W_t, defined by differentiating this family,

$$\frac{d}{dt} L_t^* g = L_t^*(W_t g) \tag{1.13}$$

is a smooth curve of linear vector fields – i.e. is a combination of the $z_i\partial_{z_j}$ with coefficients depending smoothly on t. Thus we can lift L_t to a family of diffeomorhisms, $\tilde{L}_t$, 'upstairs' generated in the same way by the lifts V_t of the W_t.

Thus we are reduced to the case that $F_*(0) = \mathrm{Id}$ as well as $F(0) = 0$. Then, in a possibly smaller neighbourhood U of 0, F itself is connected to the identity by a curve of diffeomorphisms (onto their images) fixing 0 and with differential Id their. Namely,

$$F(z)_i = z_i + \sum_{jk} a_{ijk}(z) z_j z_k,\ F_t(z) = z_i + t \sum_{jk} a_{ijk}(z) z_j z_k,\ t \in [0,1]. \tag{1.14}$$

Now the same argument applies, showing that $\frac{d}{dt}F_t^*(g) = F_t^*(V_t g)$ where V_t vanishes at 0 (and in fact vanishes to second order at 0) so lifts to a curve of diffeomorphisms with end point $\tilde{F}$. Of course away from $r = 0$ $\tilde{F}$ is unique and known to exist anyway. $\square$

So, one reason to say 'blow up the origin' instead of 'introduce polar coordinates around the origin' is that it draws attention to this coordinate invariance. In fact another way of saying this is that the blow-up of a point p in a manifold M is well defined – it is a new manifold with a blow down map which is smooth

$$\beta : [M, \{p\}] \longrightarrow M. \tag{1.15}$$

Invariantly one can take $[M; \{p\}]$ – which is M with p blown up – to be $(M \setminus \{p\}) \cup (T_pM \setminus \{0\})/\mathbb{R}_+$. The claim is that this has a unique $\mathcal{C}^\infty$ structure as a manifold with boundary, where the first part is the interior and the second part, which is just a sphere, (written out invariantly as the quotient of the complement of the origin in a vector space by the radial action – if you like it is the space of half-lines through the origin) is the boundary, such that in local coordinates near p this reduces to exactly the local picture we had above. To see this, just think how the differential F_* acts on the sphere.

3. Projective coordinates

There are other ways of looking at the blow up of a point which are helpful, especially in computations. I did not do this in the lectures but here is a brief description. First of all, what are coordinates on the sphere – clearly this is involved here. Well, if we introduce the homogeneous functions on the sphere $(\mathbb{R}^n \setminus \{0\})/\mathbb{R}_+$ which are the

$$\omega_j = \frac{z_j}{r},\ r = \left(z_1^2 + \cdots + z_n^2\right)^{\frac{1}{2}} \tag{1.16}$$

then $\sum\limits_j \omega_j^2 = 1$ and

$$\sum_j \omega_j d\omega_j = 0 \text{ on } \mathbb{S}^{n-1}. \tag{1.17}$$

So, we can get local coordinates at any point on the sphere by choosing $n-1$ of these provided we abide by two rules. First, don't choose one with $\omega_j = \pm 1$ at the point, since its differential is zero and it cannot be a coordinate. Secondly, choose all of the ω_j which vanish at the point, since their differentials are not dependent on any of the others! Apart from this you are free to choose as you can easily check.

So r and appropriate choice of the ω_i's give coordinates on the blown up space near each point. However, such 'polar coordinates' are not so easy to compute with. Instead one can use the corresponding projective coordinates at the point. At least one of the ω_j's is non-zero (not limiting yourself to the ones you chose as coordinates). Choose one of the corresponding z_j's, (if one $\omega_j = \pm 1$ of course it has to be that one,) as a 'radial variable' – it might be negative nearby, but no matter. Then as projective coordinates we can use z_j and the $t_k = z_k/z_j$ for $k \neq j$. As a little exercise you can check that

LEMMA 3. *Near any (boundary is the only interesting case) point of* $[0, \infty) \times \mathbb{S}^{n-1}$ *the* t_k *and one* z_j *described above give local coordinates in terms of which the* *lifts of the linear vector fields are*

$$z_l\partial_{z_s} \longmapsto \begin{cases} t_l\partial_{t_s} & l,s\neq j\\ \partial_{t_s} & l=j,\ s\neq j\\ t_k(z_l\partial_{z_l}-\sum\limits_r t^r\partial_{t_r}) & l\neq j, s=l\\ z_l\partial_{z_l}-\sum\limits_r t^r\partial_{t_r} & l=s=j.\end{cases} \tag{1.18}$$

So one can certainly cover the blow-up by patches in which such projective coordinates are valid.

4. Vector bundles

The blow up of a point in a manifold, as described above, is coordinate invariant. For a real vector bundle $E\longrightarrow M$ over a manifold M the zero section is a submanifold of E which is diffeomorphic to M but is just given by a point in each fibre. It follows that we can blow up each point 'of M' (thought of as the zero section) in the corresponding fibre and more significantly that the fibres will fit together smoothly as the point varies.

PROPOSITION 1. *For a real vector bundle $E\longrightarrow M$ the set $[M;0_E]=(E\setminus 0_E)\cup(\mathbb{S}E)$ where $\mathbb{S}E\longrightarrow M$ is the bundle of spheres $(E\setminus 0_E)/\mathbb{R}_+$, has a natural structure as a manifold with boundary and smooth blow-down map*

$$\beta:[E;0_E]\longrightarrow E \tag{1.19}$$

which restricts to the blow-down map for $[E_p;\{p\}]$ for each $p\in M$, and is consistent with local trivializations of E over open sets of M.

PROOF. I will not dwell too much on this although it is important. Taking a trivialization of E over an open set U identifies everything with a product $U\times[\mathbb{R}^n;\{0\}]$ and everything is seen to make sense as stated. A change of trivialization is, on the overlap in the bases, a smooth family of linear maps on the fibres. The discussion above shows that this lifts to a smooth family of maps on the fibres of the blown up spaces proving the result, but one should do it more carefully than I am. □

The preimage of 0_E under the blow-down map is the 'front fact' of the blown up space – in this case it is diffeomorphic to the sphere bundle of E.

Note that bundle isomorphisms $E\longrightarrow F$ lift to diffeomorphisms of the blown up spaces $[E,0_E]\longrightarrow[F;0_F]$ by the same arguments as above (although general smooth bundle maps do not – they do not 'know where to go'). What is more important in the sequel is that general smooth diffeomorphisms preserving the zero section also lift smoothly.

LEMMA 4. *If $E\longrightarrow M$ is a vector bundle and $U_1,U_2\supset 0_E$ are open neighbourhoods of the zero section with $F:U_1\longrightarrow U_2$ a diffeomorphism such that $F(0_E)=0_E$, then F lifts to a diffeomorphism between neighbourhoods of the front face of $[M;0_E]$.*

PROOF. Time is short so I will not go through this in detail. It can be proved in a way that is quite close in spirit to the proof of Lemma 2 above proceeding in steps. First, a diffeormorphism of M lifts to a diffeomorphism of E which is the identity on the fibres. These diffeomorphisms lift to the blown up space and hence we can assume that F is actually the identity on 0_E. The differential of F at the zero section is then the identity on tangent vectors to 0_E and hence projects to a bundle isomorphism of E. Again this lifts, so this projection can also be arranged to be the identity. It then follows by a partition of unity argument that F can be connected to the identity through a smooth family of diffeomorphisms which all have these two properties. Again these are given by integration of a one-parameter family of vector fields which vanishes at 0_E. In local trivializations it is easy to see that such a vector field lifts to be smooth – using the arguments above – and then the integration can be done on the blown-up space to construct the lifted diffeomorphism.

Alternatively you can sit down and compute the lift in local coordinates. It is not all that hard. □

5. Embedded submanifolds

Now the final step, for the moment, is to show that if $Y \subset M$ is an embedded submanifold of another manifold then there is a well defined blown-up manifold with boundary $[M;Y]$ which is such that in local coordinates in which Y is given by the vanishing of the first k coordinates then $[M;Y]$ is just the product of the blow up of the origin in these variables with the coordinate space in the other variables. One way to see this without doing too much work is to use Lemma 4 and the collar neighbourhood theorem. The latter shows that for an embedded submanifold there is always a diffeomorphism of a neighbourhood of Y in M to the total space E of a vector bundle over M such that Y is mapped to the zero section. This in fact characterizes the condition that the submanifold be embedded. The vector bundle in question is the normal bundle to Y in M, the quotient $T_Y M/TY$. From this the existence of the blown up manifold with boundary, as $[M;Y] = (M \setminus Y) \cup (TY \setminus 0_Y)/\mathbb{R}_+$ with a natural $\mathcal{C}^\infty$ structure and blow-down map

$$\beta : [M;Y] \longrightarrow M \tag{1.20}$$

follows. It has the 'obvious properties' being a diffeomorphism of the interior onto $M \setminus Y$ and restricting to the boundary, which is the front face $\mathbb{S}Y = (TY \setminus 0_{TY})/\mathbb{R}_+$ as the projection to Y.

The generalization of the discussion above of vector fields is

PROPOSITION 2. *Under the blow up of an embedded submanifold Y of a manifold M the smooth vector fields on M which are tangent to Y lift under the blow down map (are related under it) to unique smooth vector fields on $[M;Y]$. The lifted vector fields span, over $\mathcal{C}^\infty([M;Y])$ all smooth vector fields on $[M;Y]$ which are tangent to the boundary – the front face produced by the blow up of Y.*

6. Projective blow up

In projective blow up, we simply use 'two-sided' polar coordinates. In other words instead of the polar coordinate map (1.3) we use the closely related map

$$\beta_{\mathbb{P}} : \mathbb{R} \times \mathbb{P}^2 \ni (\rho, \eta) \longmapsto \rho\eta \in \mathbb{R}^2,\ \mathbb{P}^2 = \mathbb{S}^2/\pm\,. \tag{1.21}$$

This is still smooth and surjective and is locally near each point 'the same map'. The advantage is that there is no boundary on the left. The disadvantage (not very serious generally) is that $\mathbb{P}^2$ is not orientable. One can go ahead and check that projective blow up is indeed globally well-defined as for the radial case. The relationship between them is pictured here:

$$\begin{tikzcd}[M;Y] \equiv [[M;Y]_{\mathbb{P}};H] \arrow[r, "\beta(H)"] \arrow[dr, "\beta(Y)"'] & {[M;Y]_{\mathbb{P}}} \arrow[d, "\beta_{\mathbb{P}}(Y)"] \\ & M.\end{tikzcd} \tag{1.22}$$

Here H is the hypersurface $\{\rho = 0\} \subset [M;Y]_{\mathbb{P}}$ which is the inverse image of Y under the projective blow up. Thus, radial blow up factors through projective blow up and in that sense the latter is more 'fundamental'.

Why not use projective blow up? There are at least two reasons. One is that the functions we deal with often do not lift to be smooth across H under projective blow up, but 'smooth up to it from both sides' so the simiplication is only apparent. The other is that we are often dealing with boundaries in the first place and then the projective blow up does not really make sense anyway, or rather reduces to the same thing.

7. Parabolic blow up

I did not talk about this in the end. It is discussed extensively in the book on the C.L. Epstein, [**?**].

8. What does this buy us?

So what can we do with this blow up? We can resolve orbifolds and other manifolds which look like bundles of cones over a smooth manifold. We can also 'resolve' Morse functions. Suppose that M is a compact manifold, then it always carries a Morse function, a smooth function $u \in \mathcal{C}^\infty(M)$ with the property that at every point of M either the function is non-stationary, $du(p) \neq 0$, or else if $du(p) = 0$ then the Hessian is invertible, where the Hessian is the map

$$T_pM \ni v \longmapsto H_u v(p) \in T_p^*M \tag{1.23}$$

which is induced by taking a smooth vector field V on M with $V(p) = v$ and considering $d(Vu)(p) \in T_p^*M$ – which can be seen not do depend on the particular choice of V. A Morse function only has finitely many critical points $\{p_1, \dots, p_N\}$, and if these are blown up then near each of the new front faces it takes the form

$$u = u(p_i) + r_i^2 U_i \tag{1.24}$$

were U_i is smooth and has $dU_i \neq 0$ on $U_i = 0$. In particular this means that the level sets of u are all unions of smooth manifolds which meet transversally – they are resolved to normal crossings. The level set for a critical value has been 'resolved' to $r_i = 0$, the new front face, plus $U_i = 0$ which is a smooth hypersurface which is transversal to $r_i = 0$.

[Picture please!]

Let me try at this stage to anticipate some of what I will show later about such a 'resolution'. Why should such a blow up help? One thing to look at is the Lie algebra of smooth vector fields which annihilate the function u. Where $du \neq 0$ on a level set, this is just the Lie algebra of vector fields tangent to the fibres. At the singular point, on the singular stratum, it becomes much more complicated. However, after the single blow up of the critical point, as described above, the smooth vector fields annihilating u, i.e. pairing to zero with du, are locally the ones tangent to $r_i = 0$ or $U_i = 0$ away from the intersection but at the intersection we can take $U_i = s$ to be one of the coordinates, y_k the others and then the vector fields are locally spanned by

$$r\partial_r - 2s\partial_s,\ \partial_{y_k}. \tag{1.25}$$

So, this is rather degenerate, but what I want to show later is that we can 'resolve' such vector fields and as a result discuss the properties of differential operators which are in the enveloping algebra.

If we want to do more than this – resolve more complicated singular objects or objects more complicated than spaces – for instance Lie algebras of vector fields – then we need to do two things. We need to iterate blow ups, and we need to blow up submanifolds of manifolds with corners. The latter obviously will arise on iteration of blow ups since each time we blow up a new boundary hypersurface emerges and the simplest case is when these meet transversally. I will talk about both these things tomorrow, but just suppose it works out well! Then we can resolve arbitrary projective algebraic varieties (courtesy of Hironaka), we can resolve smooth actions of compact Lie groups on compact manifolds (or proper actions of compact groups). I cannot cover all these things but I will try to describe some of them and also try to give an idea of what I really mean by resolution.

9. A list of theorems!

Still to come!