**11 February, 1998**

PROBLEM SET I FOR 18.157

Due on Thursday, February 19, 1998. These are problems take from the 'Microlocal Analysis' notes (and sometimes changed a little)

*Problem* 1. Show that the function $u(x) = \exp(x)\cos[e^x]$ 'is' a tempered distribution. Part of the question is making a precise statement as to what this means!

*Problem* 2. Show, in detail, that for each $m \in \mathbb{R}$

$$(1 + |\xi|^2)^{-\frac{1}{2}m} \in S^m_\infty(\mathbb{R}^p; \mathbb{R}^n) \tag{1}$$

for any $p$. Use this to show that

$$S^m_\infty(\mathbb{R}^p; \mathbb{R}^n) \cdot S^{m'}_\infty(\mathbb{R}^p; \mathbb{R}^n) = S^{m+m'}_\infty(\mathbb{R}^p; \mathbb{R}^n).$$

*Problem* 3. Prove Borel's lemma (using the idea of asymptotic summation). That is, any sequence of complex numbers can occur as the sequence of derivatives, at the origin, of a smooth function on $\mathbb{R}$.

*Problem* 4. Show that *Weyl quantization*

$$S^\infty_\infty(\mathbb{R}^n; \mathbb{R}^n) \ni a \longmapsto q_W(a) = (2\pi)^{-n} \int e^{i(x-y)\cdot\xi} a(\frac{x+y}{2}, \xi) d\xi \tag{2}$$

is well-defined by continuity from $S^{-\infty}_\infty(\mathbb{R}^n; \mathbb{R}^n)$ and induces an isomorphism

$$S^m_\infty(\mathbb{R}^n; \mathbb{R}^n) \underset{q_W}{\overset{\sigma_W}{\rightleftarrows}} \Psi^m_\infty(\mathbb{R}^n) \ \forall\ m \in \mathbb{R}. \tag{3}$$

Find an asymptotic formula relating $q_W(A)$ to $q_L(A)$ for any $A \in \Psi^m_\infty(\mathbb{R}^n)$. (Unless I do not get far enough and have not done the relation between left and right symbols).