

**TOPICS IN ANALYSIS
SMOOTH OPERATOR ALGEBRAS AND K-THEORY**

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ABSTRACT. These are ongoing notes for Lectures on MWF in Evans 45 for the course Math 276-2: Topics in Analysis. I will add here such notes as I manage to write up as I go along and some references.

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Preface

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NOTES

- 1 Expand
- 14 Go through notes from Paul and Co.
- 32 Write Lecture 29.
- 35 Write Topic 4.
- 37 Write Topic 6.
- 38 Write Topic 7.
- 40 Write Topic 9.
- 45 Write Lecture 36.

Additional topics

- Geometric forms of determinant, determinant bundle and gerbe.
- Hopkins and Singer?
- Dirac and Bunke.
- Segal's classifying spaces
- Loop groups and representations.
- Primitive determinant bundle over whole of loop group.
- Higher Gerbes.