From Microlocal to Global Analysis

Richard Melrose

Massachusetts Institute of Technology
E-mail address: rbm@math.mit.edu

0.7I; Revised: 15-5-2008; Run: May 15, 2008

1991 *Mathematics Subject Classification.* All over the shop

Contents

Preface

This semester, Spring 2008, I am trying to get these lectures notes close to a finished form. They represent accumulated notes from various different 'Microlocal Analysis' courses and seminars at MIT. In particular in the seminar this semester, which is a continuation of a course (also run as a seminar) last semester, we hope to complete a proof of the families index theorem of Atiyah and Singer and some version of Weyl asymptotics for self-adjoint elliptic pseudodifferential operators; maybe we will also get to Fourier integral operators.

There are many people to thank, including recent participants and people who have offered corrections and suggestions:

Jacob Bernstein
Benoit Charbonneau
Kaveh Fouladgar
Austin Ford
Sine Rikke Jensen
Mark Joshi
Nikola Kamburov
Jonathan Kaplan
Chris Kottke
Edith Mooers
Vedran Sohinger
Peter Speh
Raul Tataru
Andras Vasy
Fang Wang
Lu Wang
Zuoqin Wang
Raymond Wu

Introduction

I shall assume some familiarity with distribution theory, with basic analysis and functional analysis and a passing knowledge of the theory of manifolds. Any one or two of these prerequisites can be easily picked up along the way, but the prospective student with none of them should perhaps do some preliminary reading:

Distributions: A good introduction is Friedlander's book [**6**]. For a more exhaustive treatment see Volume I of Hörmander's treatise [**10**].

Analysis on manifolds: Most of what we need can be picked up from Munkres' book [**11**] or Spivak's little book [**14**].