

18.158, Spring 2013: Analysis on loop spaces

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ABSTRACT. Lecture notes written progressively during Spring 2013.

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Introduction

In this course I will discuss analysis and geometry of loop spaces. These are particular, but as it turns out very special, examples of Fréchet manifolds. One thing I will *not* try to do is discuss analysis in general Fréchet manifolds – for one thing there is very little to discuss! Nevertheless, let me first describe these and try to show what the problem is. Then I will go on to give an overview of loop spaces, why they are of current interest and where I hope to get to by the end of the semester.

Prerequisites:- I will assume familiarity with standard theory of smooth manifolds, but will try to discuss anything substantial that is used. The same applies to Fréchet spaces, groups, operator algebras and the like.

