18.155 LECTURE 23 6 DECEMBER, 2016

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ABSTRACT. Elliptic operators on manifolds

Before lecture

- Diffeomorphism invariance of Sobolev spaces
- Partitions of unity on (compact) manifolds
- Sobolev spaces on manifolds
- Linear differential operators
- Symbols and ellipticity
- Elliptic operators are Fredholm
- Laplace Beltrami operator
- deRham and Hodge Theorems

AFTER LECTURE

Three big theorems dealing with differential operators to understand!

(1)

Theorem 1 (deRham). On any compact C^{∞} manifold the deRham cohomology groups (i.e. linear spaces)

(1)
$$H^{k}_{\mathrm{dR}}(M) = \left\{ u \in \mathcal{C}^{\infty}(M; \Lambda^{k}); du = 0 \right\} / d\mathcal{C}^{\infty}(M; \Lambda^{k-1})$$
$$\xrightarrow{\simeq} \left\{ u \in \mathcal{C}^{-\infty}(M; \Lambda^{k}); du = 0 \right\} / d\mathcal{C}^{-\infty}(M; \Lambda^{k-1})$$

are naturally isomorphic with the singular (or simplicial or $\check{C}ech$) cohomology groups with complex coefficients.

(2)

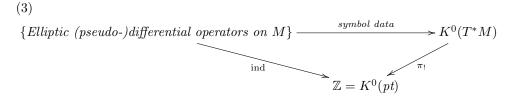
Theorem 2 (Hodge). On any compact C^{∞} manifold equipped with a Riemann metric the space of harmonic forms, the null space of the Laplace-Beltrami operator

(2)
$$H^k_{Ho}(M) = \left\{ u \in \mathcal{C}^{\infty}(M; \Lambda^k); \Delta u = 0 \right\} \xrightarrow{\simeq} H^k_{dR}(M).$$

(3)

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Theorem 3 (Atiyah-Singer). On any compact C^{∞} manifold there is a commutative diagram



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