18.155 LECTURE 10
OCTOBER 13, 2016

RICHARD MELROSE

Abstract. Notes before and then after lecture.

Read: Notes Chapter 2.

Before lecture

I will discuss Hilbert spaces. I'm assuming you have some familiarity with this – if not let me know and I may be able to slow down. Otherwise I will go fairly quickly through the basic treatment. We are mostly interested in infinite-dimensional separable Hilbert spaces. The non-separable case is not so different but everything needs to be stated carefully since there is no orthonormal basis.

I am not expecting to cover all this in one lecture but don't want to take too much time over it.

- Hermitian inner products, Cauchy-Schwarz inequality, (pre-)Hilbert spaces
- Parallelogram law, necessity
- Gramm-Schmidt, separability, orthonormal bases.
- Bessel's inequality, Fourier-Bessel expansion, l^2.
- Hermite polynomials and orthonormal basis of $L^2(\mathbb{R}^n)$.
- Convexity Lemma, projections, Riesz' Representation Theorem
- Compact sets, compact inclusions, Sobolev spaces
- The algebra of bounded operators, adjoints
- Group of invertible operators, unitary subgroup, isometric isomorphisms, Kuiper's theorem (without proof).
- Finite rank and compact ideals
- Hilbert-Schmidt and and trace class ideals
- Spectral theorem, functional calculus, compact case.
- Fredholm and semi-Fredholm operators. Generalized inverses, index.

Department of Mathematics, Massachusetts Institute of Technology
E-mail address: rbm@math.mit.edu