This anonymous quiz is just to help me judge the level at which I should begin. Please put an A, B or C in the margin on the left next to each question, where

A Means that you know the answer straight away or how to prove the statement.
B Means that you believe you could work it out in five or ten minutes.
C Means that you suspect you don’t know some necessary underlying results or do not understand the statement.

Note that there are several statements here that I expect you not to know or understand.

1. Let $C_0([-N, N])$ be the space of continuous functions on $\mathbb{R}$ which vanish outside $[-N, N] \subset \mathbb{R}$. Let $C^\infty(\mathbb{R})$ be the space of bounded continuous functions on $\mathbb{R}$ with the supremum norm. Is the union $\bigcup_N C_0([-N, N])$ dense in $C^\infty(\mathbb{R})$?

2. Let $C_0([0, 1])$ be the space of continuous functions on $[0, 1]$ with supremum norm. Are there any continuous linear functionals $u : C_0([0, 1]) \rightarrow \mathbb{C}$ such that $u(fg) = u(f)u(g)$ for all $f, g \in C_0([0, 1])$, where $fg(x) = f(x)g(x)$?

3. Let $L^1([0, 1])$ and $L^2([0, 1])$ be the Lebesgue spaces on $[0, 1]$. What exactly is an element of each of these spaces? What are their standard norms?

4. Which of $L^1([0, 1])$ and $L^2([0, 1])$ is a Hilbert space?

5. What are all the continuous linear functionals $u : L^1([0, 1]) \rightarrow \mathbb{C}$ such that $u(fg) = u(f)u(g)$ for all $f, g \in L^2([0, 1])$?

6. Let $u : \{ (x, y); x^2 + y^2 < 1 \} \rightarrow \mathbb{C}$ be a once differentiable function on the open unit ball which satisfies

$$\frac{\partial u}{\partial x} + i \frac{\partial u}{\partial y} = 0 \text{ in } x^2 + y^2 < 1.$$ 

Why is it true that $u$ is infinitely differentiable?

7. What functions are there as in the previous question which satisfy in addition $\frac{\partial^k u}{\partial x^k}(0, 0) = 0$ for all $k$?

8. Every twice differentiable solution of the wave equation in two variables, $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$ in $\mathbb{R}^2$ is of the form $u_1(x + t) + u_2(x - t)$ for two twice differentiable functions of one variable.

9. There is no smooth map $f : \mathbb{R} \rightarrow \mathbb{R}^2$ which is surjective.

10. For any sequence of real numbers $a_j, j = 0, 1, \ldots$, there is a smooth function $u : \mathbb{R} \rightarrow \mathbb{R}$ such that $\frac{d^j u}{dx^j}(0) = a_j$ for all $j$. 

Department of Mathematics, Massachusetts Institute of Technology
E-mail address: rbm@math.mit.edu