DO YOU HAVE THE BACKGROUND TO TAKE 18.102?

Some of you may be wondering whether you have the background to take the course. You can ask me and we can chat about it. Here are some pointers to help you decide.

I will use the theory of metric spaces quite freely. If you don't know *anything* about them then you need to read up, and if you do not have the time, inclination or current knowledge to do so, then you should be taking 18.100B and probably leave this course until later.

Here is an example of what I mean by casual use of the theory of metric spaces. In the first lecture I 'reminded you' that for maps between two metrics spaces $f : X \longrightarrow Y$ continuity can be described in three (and more of course) equivalent ways.

(1) If $x_n \to x$ is any convergent sequence in X then $f(x_n) \to f(x)$ converges in Y.
(2) If $O \subset Y$ is open in Y then $f^{-1}(O) \subset X$ is open in X.
(3) If $C \subset Y$ is closed in Y then $f^{-1}(C) \subset X$ is closed in X.

You could have various problems with this. If you don't know what convergence of sequences, or open/closed sets are then you do not know much about metric spaces. Maybe you do not know the notation $f^{-1}(O)$ for a map $f : X \longrightarrow Y$ with $O \subset Y$. By definition it is the 'preimage' of O under f, consisting precisely of *all* the points in X which are mapped into O by f,

$$f^{-1}(O) = \{x \in X; f(x) \in O\}.$$

Again if you don't know about this it is not a good sign – all correctable of course but if there is too much data missing you are in trouble.

So, what about the proof of the equivalence of these three conditions? As I say, this is standard metric space theory. The last two are 'manifestly equivalent' because the complement of an open set is closed and the complement of a closed set is open. Furthermore, if $O \subset Y$ then $f^{-1}(Y \setminus O) = X \setminus f^{-1}(O)$ (Why? You are supposed to be in a position to work this out reasonably quickly. One can just say 'under a map every point must go somewhere, so each point of X is mapped either into O or into its complement but not both). Now, from these the equivalence of the second two conditions should be 'obvious' which I do not normally say – I mean it is straightforward to check.

Okay, what about the equivalence of the first and second, since that is all we need to do (but you might want to think about the direct equivalence of the first and third even though this follows of course).

Suppose f has the first property, why does it have the second? Take O open in Y we want to show that $f^{-1}(O)$ is open in X using the convergence property. This means that for each point $p \in f^{-1}(O)$ (if there are any, but if it is empty then it is open by *fiat*) there must be an $\epsilon > 0$ such that $d(x, p) < \epsilon$ implies $x \in f^{-1}(O)$ – it has to contain an open ball around each point. The *opposite* statement is that for each $k > 0$ there is a point $x_k \in X$ with $d(p, x_k) < 1/k$ but $f(x_k) \notin O$. This however means that $x_k \to p$ so $f(x_k) \to f(p) \in O$ but we know that O is open so

$f(x_k) \in O$ for k large enough, which is a contradiction. So in fact $f^{-1}(O)$ must be open.

Conversely, suppose that the second condition holds and we want to check the first. This is easier. Suppose $x_n \to x$ in X then for a given $\epsilon > 0$ the ball $O = \{y \in Y; d(y, f(x)) < \epsilon\}$ is open, so $f^{-1}(O)$ is open, by assumption. Since $x_n \to x$ for large enough n it follows that $x_n \in f^{-1}(O)$ which means that $f(x_n) \in O$ which implies that $f(x_n) \to f(x)$.

If this is all nonsense you better think carefully about taking the course and maybe we should have a chat. However, just because it takes you a while to understand things does *not* mean you have a (serious) problem.

Of course I will use quite a lot more than this – uniform convergence, behaviour of compact sets, etc. If you don't know it the question is whether you can handle it in the time available.