TEST 2 FOR SECTION 1 OF 18.100B/C
TO BE SUBMITTED BY 5PM TUESDAY NOVEMBER 23.

NAME:____

This is a take-home test. You should not talk to anyone else about the questions or consult written, on-line or other material including your own notes. I do not want time to be a major concern here so you can have up to two hours in a single block after first reading the questions to complete your answers. If for some reason you depart from these conditions you should fully describe the circumstances under which you took the test.

The completed test can be dropped off in 2-108 or submitted electronically to rbm at math dot mit dot edu.

(1) If $f : X \longrightarrow Y$ is a continuous map between metric spaces and $K \subset X$ is compact, prove that $f(K) \subset Y$ is compact using the definition of compactness through open covers.

NAME:____

(2) Suppose $f : [0, 1] \longrightarrow \mathbb{R}$ satisfies

$$|f(x) - f(y)| \leq |x - y|^{\frac{3}{2}} \ \forall \ x, y \in [0, 1].$$

Explain why it follows that $f(0) = f(1)$.

NAME:____

(3) If $\alpha : [0,1] \longrightarrow \mathbb{R}$ is given by

$$\alpha(x) = \begin{cases} x-1 & 0 \leq x < \frac{1}{2} \\ x+1 & \frac{1}{2} \leq x \leq 1 \end{cases}$$

and $f = 2x$, explain why the Riemann-Stieltjes integral $\int_0^1 f d\alpha$ exists and compute its value, justifying your arguments carefully.