HOMEWORK 8 FOR 18.100B/C, FALL 2010
DUE FRIDAY 12 NOVEMBER

Since Thursday November 11 is a holiday, homework is due in 2-108 by 11AM on Friday 12 November, or by email before 5PM on the same day.

(1) Rudin Chap 6 No 2. Suppose $f : [a, b] \longrightarrow \mathbb{R}$ is continuous and non-negative, $f(x) \geq 0$ for all $x \in [a, b]$. Show that if $\int_a^b f(x)dx = 0$ then $f(x) = 0$ for all $x \in [a, b]$.

(2) Rudin Chap 6 No 4. Suppose $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = 0$ when x is irrational and $f(x) = 1$ when x is rational. Show that $f \notin \mathcal{R}$ (the space of Riemann integrable functions) on any interval $[a, b]$ where $a < b$.

(3) Rudin Chap 6 No 5. Suppose $f : [a, b] \longrightarrow \mathbb{R}$ is bounded and that $f^2 \in \mathcal{R}$ does it follow that $f \in \mathcal{R}$? What if $f^3 \in \mathcal{R}$?

(4) Rudin Chap 6 No 8. Suppose that $f : [1, \infty) \longrightarrow \mathbb{R}$ is non-negative and monotonic decreasing. Show that $\lim_{b\to\infty} \int_1^b f(x)dx$ exists (and is finite) if and only if $\sum_{n=1}^{\infty} f(n)$ exists (and is finite).

(5) Suppose that $\alpha : [a, b] \longrightarrow \mathbb{R}$ is monotonic increasing and $f \in \mathcal{R}(\alpha)$ is real-valued and Riemann-Stieltjes integrable on $[a, b]$. Show that for every $\epsilon > 0$ there is a continuous function $g : [a, b] \longrightarrow \mathbb{R}$ such that $\int_a^b |f - g| d\alpha < \epsilon$. For a hint, see Rudin Chap 6 No 12.