HOMEWORK 2 FOR 18.100B/C SECTION 1, FALL 2010
DUE THURSDAY, SEPTEMBER 23 AT 11 IN 2-108 OR BY 5PM ELECTRONICALLY.

HW2.1 Rudin Chap 1, Prob 13: If x and y are complex numbers show that

$$||x| - |y|| \leq |x - y|.$$

HW2.2 Let (X, d) be a metric space – i.e. d is a metric on X. Define the (British Railway) 'metric' on X by choosing a point $L \in X$ and defining

(a) $d_{\mathrm{BR}}(x, x) = 0, \ \forall \ x \in X$.

(b) $d_{\mathrm{BR}}(x, y) = d(x, L) + d(L, y)$ if $x \neq y$.

Show that this is indeed a metric and that every subset of X not containing L is open. (Note that L represents London!)

HW2.3 Rudin Chap 2, Prob 4. Is the set of irrational numbers countable? (Justify your answer!)

HW2.4 Rudin Chap 2, Prob 5. Construct a bounded subset of $\mathbb{R}$ which has exactly three limit points.

HW2.5 In each case determine whether the given function d is a metric (Prove your answer!).

(a) Fix $N \in \mathbb{N}$. Let X be the set of sequences of zeroes and ones of total length N. For two sequences x, y let $d(x, y)$ be the number of places at which the two sequences differ.

(b) $X = \mathbb{R}$, $d(x, y) = (x - y)^2$.

(c) $X = \mathbb{R}$, $d(x, y) = \sqrt{|x - y|}$.

(d) $X = \mathbb{R}$, $d(x, y) = |x - 2y|$.

(e) $X = \mathbb{R}$, $d(x, y) = \frac{|x-y|}{1+|x-y|}$.