

**HOMEWORK 1 FOR 18.100B/C, FALL 2010**  
**SECTION 1 (MELROSE)**  
**DUE 11AM THURSDAY, 16 SEPTEMBER IN 2-108.**

Blurb: Paper homework is due on Thursdays in 2-108 by 11AM. There will be a tray to put it in and I will remove it after lecture. No paper homework will be accepted late, period. HOWEVER, you may do your homework with TeX (I encourage you to do this, preferably using LaTeX) or scan your homework to a pdf file and email it to me (rbm at math dot mit dot edu works best). The deadline for scanned homework is 5PM on the Thursday on which it is due. Late homework will be accepted, and will be graded, but will in general incur a substantial penalty.

Suggestion. First think through how you will do a problem and then try to write down your argument as clearly and briefly as you can. The objective is to convince me that you really do understand why the statement is true, not to right a thesis just yet. It is always a good idea to think of your (possible) audience when writing. In this case it is ME, maybe it will be a grader next week but I will still look at your work. So, ask yourself when you write down an argument – Is this really convincing? Are the gaps too large? Are there missing steps? Is all this obvious stuff really necessary? Try to find the right level for your target audience – in this case it is clear who it is.

The rational numbers are  $\mathbb{Q}$ , the real numbers  $\mathbb{R}$  and the complex numbers  $\mathbb{C}$ .

HW1.1 Rudin Chap 1, Prob 1. If  $r$  is rational and non-zero and  $x$  is irrational show that  $r + x$  and  $rx$  are irrational.

Hint: An irrational number is an element of  $\mathbb{R}$  which is not in  $\mathbb{Q}$ . There are only two possibilities, that  $r + x$  is rational or irrational. By assumption  $r$  is rational. If  $r + x$  were rational then  $x$  would also be rational (why?) . . . . The second part is similar.

HW1.2 Show that there is no rational number  $r = m/n$  such that  $r^3 = 3$ .

Hint: Suppose there is such a rational number  $r$ , choose a ‘presentation’  $r = m/n$  where the integers  $m$  and  $n$  do not both have a factor of 3 in their prime decompositions and deduce a contradiction.

HW1.3 Rudin Chap 1, Prob 4. Let  $E$  be a nonempty subset of an ordered set; suppose  $\alpha$  is a lower bound for  $E$  and  $\beta$  is an upper bound. Prove that  $\alpha \leq \beta$ .

Hint: Look at the order relations between  $\alpha$ ,  $\beta$  and some element  $\gamma \in E$ .

HW1.4 Show that the real numbers of the form  $q_1 + q_2\sqrt{3}$ , where  $q_1$  and  $q_2$  are rational, form a field. Hint: First check that the negative and inverse of a non-zero number of this form is of the same form, then check that the sum and product of two of these numbers is also of the same form. Then go through the list of axioms of a field checking that they are true because they are true for the reals.

HW1.5 Rudin Chap 2, Prob 2: A complex number  $z$  is said to be *algebraic* if there are integers  $a_i$ ,  $i = 0, \dots, n$ , not all zero, such that

$$a_0 z^n + a_1 z^{n-1} + \dots + a_{n-1} z + a_n = 0.$$

Assuming the easy part of the fundamental theorem of algebra, that such an equation can have at most  $n$  distinct solutions, prove that the set of all algebraic numbers is countable. Rudin's hint: For every positive integer  $N$  there are only finitely many such equations with

$$n + |a_0| + |a_1| + \dots + |a_n| = N.$$

Using this write the algebraic numbers as a countable union of finite sets.