18.100B, FALL 2002
PRACTICE TEST 2

Try each of the questions; they will be given equal value. You may use theorems from class, or the book, provided you can recall them correctly!

Problem 1

Let $f : [0,1] \longrightarrow \mathbb{R}$ be a continuous real-valued function. Show that there exists $c \in (0,1)$ such that

$$\int_0^1 f(x)dx = f(c).$$

Problem 2

(This is basically Rudin Problem 4.14)

Let $f : [0,1] \longrightarrow [0,1]$ be continuous.

(1) State why the the map $g(x) = f(x) - x$, from $[0,1]$ to $\mathbb{R}$ is continuous.
(2) Using this, or otherwise, show that $L = \{x \in [0,1]; f(x) \le x\}$ is closed and $\{x \in [0,1]; f(x) < x\}$ is open.
(3) Show that L is not empty.
(4) Suppose that $f(x) \neq x$ for all $x \in [0,1]$ and conclude that L is open in $[0,1]$ and that $L \neq [0,1]$.
(5) Conclude from this, or otherwise, that there must in fact be a point $x \in [0,1]$ such that $f(x) = x$.

Problem 3

Consider the function

$$f(x) = \frac{-x(x+1)(x-100)}{x^{44} + x^{34} + 1}$$

for $x \in [0, 100]$.

(1) Explain why f has derivatives of all orders.
(2) Compute $f'(0)$.
(3) Show that there exists $\epsilon > 0$ such that $f(x) > 0$ for $0 < x < \epsilon$.
(4) Show that there must exist a point x with $f'(x) = 0$ and $0 < x < 100$.

Problem 4

If $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$ are two functions which are continuous at 0, show that the function

$$h(x) = \max\{f(x), g(x)\}, \ x \in \mathbb{R}$$

is also continuous at 0.