18.155 LECTURE 25
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This is intended to give some idea of what I (or I hope we) will cover in 18.966 next semester and, for me, preparation for the two lectures I will give at Princeton tomorrow – especially the first.

(1) Surface, S, metrics, conformal structures.
(2) $\mathcal{M}(S)$ conformal structures up to orientation-preserving diffeomorphism.
    We want to work on this space – really an algebraic stack. If S is not the sphere or torus then $\mathcal{M}(S)$ has a natural metric, G_{WP}.

    Theorem 1 (Gell-Redman, M.). *The space of L^2 harmonic forms on $\mathcal{M}(S)$ is isomorphic to the cohomology of the Deligne-Mumford compactification $\overline{\mathcal{M}}(S)$.*

    Really we work in the pointed case.
(3) Complex structure, $\overline{\partial}$, integrability
(4) Complex structures, with base point, $\mathcal{J} = \{J \in \mathcal{C}^\infty(S; \Lambda^{0,1} \otimes T^{1,0}); |J| < 1\}$.
(5) Infinitesmal deformation, $T^*\mathcal{M}(S) = \{q \in \mathcal{C}^\infty(S; \Lambda^{2,0}); \overline{\partial} q = 0\}$ 'holomorphic quadratic differentials'.
(6) In each conformal class of metrics on S there is a unique metric of constant Gaussian curvature -1.
(7) The Weil-Petersson metric.
(8) Smooth surgery – changind the genus; Lefschetz maps.
(9) Uniform behaviour of the canonical metric
(10) Weil-Petersson-type metrics.

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