

**18.155 LECTURE 24**  
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Dirac operators and the heat kernel

- (1) Dirac-type operators,  $\tilde{D} \in \text{Diff}^1(M; V)$ ,  $V$  Hermitian,  $M$  Riemannian,  $\tilde{D}$  symmetric and such that  $\sigma_2(\tilde{D}^2)(\xi) = |\xi|_g^2$  is ‘Laplacian-like’.
  - $d + \delta$
  - $\bar{\partial} + \bar{\partial}^*$  on a Riemann surface
  - ‘The’ Dirac operator for a spin structure.
- (2) Clifford algebra and Clifford module structure on  $V$ .
  - Periodicity
  - Parity operator,  $\dim M = 2k$ .
  - Spin representations.
  - $\mathbb{Z}_2$ -grading
- (3) Spectral theory of  $\tilde{D}^2$ .
  - Heat kernel
  - McKean-Singer formula.

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