DIAGNOSTIC 'TEST' FOR 18.155, FALL 2013
NO MARKS FOR SUCCESS OR PENALTIES FOR ERRORS

Note: This is due on Friday September 6 – by email only: rbm AT math.mit.edu. I will look at these on Saturday morning.

I don't particularly want you to work on this – although of course you can do so. The first problem set, due next week, will be up this evening. What I would like is an honest answer indicating how much you know about each question. You are encouraged to say so when you don't know about something and I will try to compensate for this in lectures.

When you can do so, please answer these questions succinctly – just a paragraph or two at most for each. Try to be short and clear even if not complete and even a simple yes you know or know you don't is helpful (so I have left a little space).

(1) Explain why the space of continuous complex-valued functions on $\{z \in \mathbb{R}^n; |z| \le 1\}$ is a Banach space with respect to the supremum norm.

(2) Is the space of bounded functions which are continuous on $\{z \in \mathbb{R}^n; |z| < 1\}$ a Banach space with respect to the supremum norm?

(3) F. Riesz' theorem states that any continuous linear functional on a Hilbert space is given by pairing with an element of the Hilbert space. How is this proved?

(4) A function $f : \mathbb{R}^n \longrightarrow \mathbb{C}$ is said to be differentiable at $\bar{x} \in \mathbb{R}^n$ if there is a linear function $l : \mathbb{R}^n \longrightarrow \mathbb{C}$ such that for every $\epsilon > 0$ there exists $\delta > 0$ for which

$$|f(x) - l(x)| \le \epsilon |x - \bar{x}| \ \forall \ |x - \bar{x}| < \delta.$$

Show that f then has partial derivatives at $\bar{x}$.

(5) A function $g : \mathbb{R}^n \longrightarrow \mathbb{R}$ is Lebesgue measureable if $g^{-1}(a, b)$ is Lebesgue measureable for all $a < b$ in $\mathbb{R}$. If g is also bounded, explain why it is then locally Lebesgue integrable.