

## Information on 18.785, Number theory I, Fall 2023

**Website:** <https://canvas.mit.edu/courses/21658>

**Instructor:** Bjorn Poonen (he, him, his), 2-243, 617-258-8164. It is fine to address me simply as Bjorn (or Professor Poonen if you prefer).

**Office hours:** See my website.

**Email:** [poonen@math](mailto:poonen@math.mit.edu) (add [.mit.edu](mailto:poonen@math.mit.edu) if off-campus).

Math is much easier to explain in person than in email, so if you have a math question, come to office hours! If you do email me, please include your full name (at least the first time) if it will not otherwise be obvious to me who you are!

**Topics:** Dedekind domains, unique factorization of ideals, splitting of primes. Lattice methods, finiteness of the class group, Dirichlet's unit theorem. Local fields, ramification, discriminants. Zeta and L-functions, analytic class number formula. Adeles and ideles. Statements of class field theory and the Chebotarev density theorem. The adelic point of view will be emphasized.

**Prerequisites:** 18.705 (commutative algebra) is a corequisite, meaning that you should be taking it concurrently if you have not taken it already. In fact, we will need several topics from 18.705 before they are covered in 18.705 this semester (see "How to prepare" below), so students who have not seen them should read ahead in the 18.785 book. You should already have completed 18.705's prerequisites: 18.100 (real analysis), and 18.701 and 18.702 (undergraduate algebra, including Galois theory). As usual with graduate subjects, familiarity with other topics, such as undergraduate-level topology and complex analysis, will be needed at times.

**Books:** We will roughly follow a number theory textbook that Andrew Sutherland and I have been writing. Here are a few other good books that may be of use to you during the semester:

- Cassels & Fröhlich, *Algebraic number theory*, Academic Press, 1986 reprint of 1967 original.
- Koblitz, *p-adic numbers, p-adic analysis, and zeta-functions*, 2nd edition, Springer-Verlag.
- Lang, *Algebraic number theory*, 2nd edition, Springer-Verlag.
- Milne, course notes on *Algebraic number theory* and *Class field theory* at <http://www.jmilne.org/math/CourseNotes/>
- Neukirch, *Algebraic number theory*, Springer-Verlag.
- Serre, *Local fields*, Springer-Verlag.
- Serre, *A course in arithmetic*, Springer-Verlag.

Many aspects of number theory are based on commutative algebra, so at some point you will likely want to consult one or more commutative algebra texts. Here are a few excellent ones:

- Atiyah and MacDonald, *Introduction to commutative algebra*, Westview Press, 1994. This is a classic, covering many of the most important topics in a very efficient way, in 128 pages. It also is full of exercises.
- Eisenbud, *Commutative algebra*. This is a more comprehensive text, including a lot of commentary: 785 pages!
- *The stacks project*, chapter on commutative algebra. This is intended as a reference, not as a textbook to learn from.

There are many complex analysis textbooks, one of which is

- Lang, *Complex analysis, 4th edition*, Springer, 1999. Chapters 1–6 cover much more than we need.

### How to prepare:

- Find a study group to join at [psetpartners.mit.edu](https://psetpartners.mit.edu).
- Review the prerequisites. Some commutative algebra topics that we will use early on are tensor product, localization, projective modules, integral extensions, and discrete valuation rings. We will also use basic notions of category theory, such as functors. It would help to read at least the first three chapters of the 18.785 book before class starts.
- Consider learning how to use L<sup>A</sup>T<sub>E</sub>X if you don't already know it. I recommend Grätzer, *More math into L<sup>A</sup>T<sub>E</sub>X*, Springer-Verlag, 2016.
- On the Canvas website, please do not change your notification setting for Announcements — it should be “Notify me right away”.

**Lectures:** MWF 9–10am in 2-131, from Sept 6 to Dec 13, 2023. Masks are optional at MIT; see the full policy here. Because I have asthma and because there may be students whose personal situation requires them to be more careful with covid than most, I would be grateful if you would wear a mask while in the classroom, but you are still welcome in the classroom without one. MIT people can obtain free high-quality KF94 masks from vending machines on campus. Food and drink in MIT classrooms are prohibited, as always. Attendees not officially enrolled for credit or as listener may be asked to leave if the room is too full. I recommend that you keep electronic devices off to avoid distracting other students around you (see the research mentioned in this article!) A recording of each lecture will be available in Panopto Video on Canvas several hours later.

### Assignments:

- Problem sets will be posted online in Files on Canvas.
- There will be 11 of them, due at **11:30 P.M. Eastern Time** on the following dates: Sun Sept 17, Thu Sept 28, Sun Oct 8, Sun Oct 15, Sun Oct 22, Sun Oct 29, Sun Nov 5, Sun Nov 12, Sun Nov 19, Sun Dec 3, Tues Dec 12.
- Completed assignments should be submitted on Gradescope.
- Only students expecting to be officially enrolled for credit (as an MIT student or officially cross-registering from Harvard or Wellesley) should submit assignments. If you are intending to remain a listener, instead self-grade them using the posted solutions.

- You should not expect to be able to solve every problem on your own; instead you are encouraged to discuss questions with each other and to come to office hours, so that when you submit an assignment you are pretty sure that it is complete and correct. You might learn best if you attempt as many problems as you can before meeting with your study group. **Write-ups must be done independently.** (This means that it is OK for other people to explain their solutions to you, but you must not be looking at other people’s solutions as you write your own.)
- L<sup>A</sup>T<sub>E</sub>X is preferred but not required. If writing solutions by hand, please scan them into PDFs.
- Write in complete sentences with correct grammar, using the textbook as a model.
- Solving the problem is nice, but the *real* challenge is to make your solution as clear as possible, impossible to misunderstand or misinterpret! Some writing suggestions are here.
- Each problem set ends with a fake problem called “Sources consulted”. Your “solution” to this problem should consist of the words “Sources consulted: none” OR a list of all sources consulted other than the books above and lecture notes. **This is required.** (Examples of things that should be named if consulted: a classmate, a tutor, a friend, a website, a textbook not listed above, solutions from a previous semester, etc. And remember, if you consult these, you must write your solutions afterwards while not looking at them.) For some problems, there may exist solutions of varying quality online; you are discouraged from using these; if you look at one of these, or look at solutions written by students or professors in previous semesters, you *must* mention the precise source on your assignment. Use of cheating websites such as [chegg.com](http://chegg.com) as a consumer or provider is forbidden.
- For each problem, including the “Sources consulted” problem, indicate in Gradescope which pages of your PDF it appears on, following page 3 of these instructions.
- **Late submissions will not be accepted.** They will not be graded and will receive zero credit, unless excused. The only exception is that Gradescope has a grace period, allowing submission until 11:59 P.M. on the due date; this grace period is in case of technical difficulties only — the official deadline is 11:30 P.M. Gradescope will not permit uploading of solutions at or after 11:59 P.M, and solutions may not be submitted by email. So plan to submit a little early.
- Solutions will generally be posted on Canvas at 10:00 A.M. the next day. Reading them is recommended even if you solved all the problems, since there are usually different ways to present a solution.
- Because solutions are posted, extensions will generally not be granted, even with a recommendation from S<sup>3</sup> or GradSupport. But with such a recommendation you might be *excused* from completing part or all of the assignment.
- If a correct and clearly written solution of yours was not awarded credit, please first read the posted solution set for comparison, and then submit a regrade request through Gradescope. If after submitting a regrade request, the grader denies you credit but you still feel that your solution is correct, then you may appeal to me via email.

**Grading:** Based exclusively on problem sets. No exams. Your grade does not depend on how others are doing; the percentage of students who will get an A, B, etc. is not predetermined. Applying a to-be-determined function to your numeric grade yields your letter grade.

- A numeric grade of 90% or above guarantees an A- or above.
- A numeric grade of 80% or above guarantees an B- or above.
- A numeric grade of 70% or above guarantees an C- or above.

The actual function may be more generous than this, depending on the difficulty level of the problem sets. A grade of I (incomplete) will be given only when circumstances beyond your control have prevented you from completing a minor part of the work for the course, and you have been doing passing work up until the circumstances arose; in addition, you must have notified me of the difficulty as early as possible.

**MIT help resources:** Your friendly lecturer, Mathematics Academic Services 2-110, and the MIT Division of Student Life. See this chart for an even longer list of help resources at MIT.

**If a personal or medical issue is interfering with your studies:**

- Contact your medical provider if you need medical attention.
- Please do not come to class if you are potentially contagious. *This applies even if you test negative for covid and it is just a cold, say.*
- Email me.
- If you are dealing with a personal or medical issue that is impacting your ability to complete work:
  - Undergraduates should contact a dean in Student Support Services (S<sup>3</sup>): reach out to one you have worked with before, join the virtual help queue, email [s3-support@mit.edu](mailto:s3-support@mit.edu), or call 617-253-4861.
  - Graduate students should contact a dean in GradSupport at [gradsupport@mit.edu](mailto:gradsupport@mit.edu) or 617-253-4860.

These offices are here to help you. The deans will provide you with support, and help you work with me to determine next steps. In most circumstances, undergraduates will not be excused from coursework without verification from a dean. If you have some other kind of conflict, email only me (not a dean) as far in advance as possible, and I will make a decision on how to proceed.

- Typically, if excused, you will not get a deadline extension, but your remaining scores will be averaged. If you do the problem set later on, you can self-grade it using the posted solutions and come to office hours with questions.
- In case of financial hardship, consider consulting the ARM Coalition.

*Note:* Generally, you are expected to choose a manageable courseload and to manage your time, so work in other classes is usually not a reason to be excused from coursework in this class. If in a particular week there are extenuating circumstances, you must go through a dean as above to request an exemption.

**If you need disability accommodations:** Please speak with Kathleen Monagle, Associate Dean at Disability and Access Services (DAS), ideally before the semester begins or early in the semester, by visiting 5-104 or calling 617-253-1674. If you have a disability but do

not plan to use accommodations, it is still recommended that you meet with DAS staff to familiarize yourself with the services and resources of the office. If you have already been approved for accommodations, please email a PDF copy of the accommodation letter to Sefanya Hope in Mathematics Academic Services 2-110 early in the semester or as soon as you receive it.

**Other important things:**

- It is your responsibility to email me as far in advance as possible in case of an extended absence, or in case you find yourself struggling with the course for any reason.
- If you have emergency medical information you wish to share with me, or if you need special arrangements, in case the building must be evacuated, please inform me immediately: feel free to talk to me privately after class or in my office.

**Final comment:** This is a difficult time for just about everybody, and we all have different mathematical backgrounds and living situations, so please be patient with your fellow classmates (and with me, since surely there will be technological difficulties, if not mathematical ones!) Please join study groups and help each other learn!