18.725 Algebraic geometry I, Fall 2011


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Office hours: M 10:30–11:30am, M 1:30–2:30pm, and F 4–5pm in 2-244, starting 9/8/11.

Email: poonen@math (add .mit.edu if off-campus). Math is much easier to explain in person than in email, so if you have a math question, come to office hours! Anonymous emails will probably end up in my spam filter.

Lectures: TuTh 9:30-11am in 2-135

Prerequisite: 18.901 (Topology): Actually we will mostly need just the definition of a topology, as well as basic properties of compact sets. Also 18.705 (Commutative algebra) is a corequisite, which means that if you haven’t taken it before, you should be taking it concurrently. I will assume from the beginning that students understand what localization of a commutative ring at a prime ideal is, and that they know the basic definitions of category theory (e.g., the definition of equivalence of categories), so you are advised to learn these things before the course starts if you don’t already know them. This is a difficult graduate-level subject!

Books: The only required text for the course is Hartshorne, *Algebraic geometry*, Springer. In fact, there are many excellent algebraic geometry texts. Here are a few others (not required):

- Shafarevich, *Basic algebraic geometry*, Springer. (It was originally one volume, but now has been split and expanded into two volumes.) This is written at a slightly more elementary level than Hartshorne; on the other hand, it points out many connections with other branches of mathematics.
- Mumford, *The red book of varieties and schemes*, now printed by Springer (so it’s now a yellow book). This is one of the earlier textbooks on schemes, and makes an effort to show that one does not lose geometric intuition when working with schemes.
- Eisenbud and Harris, *The geometry of schemes*, Springer. This also explains the geometric intuition well.
- Vakil, *Foundations of algebraic geometry*. These are course notes by an algebraic geometer who is also a master of exposition.

Algebraic geometry is based on commutative algebra, so at some point you will likely want to consult one or more commutative algebra texts. Here are a few excellent ones:

- Eisenbud, *Commutative algebra with a view towards algebraic geometry*, Springer.

Finally, here are some encyclopedic references. I do not recommend trying to read them cover-to-cover! But they are useful if you need to look up something.

- Grothendieck (with the assistance of Dieudonné), *Éléments de géométrie algébrique (EGA)*. This is very clearly written, and is easy to read except for the fact that it is
very long (about 1500 pages) and contains statements in greater generality than you usually need.

- *The stacks project*. This is even longer (2900 pages and growing), but it is a searchable PDF file.

**Plan for the course:** Here is a rough list of topics for this semester: affine and projective varieties, sheaves, schemes, properties of schemes and morphisms, sheaves of modules, divisors, projective embeddings, differentials, with applications to curves.

As motivation, we will introduce affine and projective varieties as sets of points over an algebraically closed field (Nullstellensatz, etc.) But then instead of finishing Chapter I, we will jump to Chapter II so that the modern language of schemes becomes familiar as early as possible. So that this is possible, we will study lots of examples along the way, to develop geometric intuition within this language.

**Homework:** Assignments are posted online. Completed assignments should be submitted online before 9:30 A.M. on the Tuesday due date. Homework submitted after solutions are posted will not be accepted. At the top of each assignment should appear either the text “Sources consulted: none” or a list of all sources consulted other than the recommended texts and your own notes from lecture. This is *required*. (Examples of things that should be listed if used: names of study group partners, Wikipedia, etc.)

You should not expect to be able to solve every single problem on your own; instead you are encouraged to discuss questions with each other or to come to office hours. If you meet with a study group, you may find it helpful to do as many problems as you can on your own beforehand. But write-ups must be done independently. (In practice, this means that it is OK for other people to explain their solutions to you, but you must not be looking at other people’s solutions as you write your own.)

Use proofs in the book as a model for the level of detail expected. Write in complete sentences whenever reasonable.

If you have questions about the homework, it is best to ask these in office hours.

**Grading:** Based exclusively on homework. No exams.

**If you get sick:** Please do not come to class. Instead contact your medical provider for medical attention, email me, and (if you are an undergraduate and will be failing to turn in an assignment) contact Student Support Services to have an official notice sent to all your instructors.

**Other important things:** It is your responsibility to inform me as far in advance as possible in case of an extended absence, or in case you find yourself struggling with the course for any reason. If you need disability-related accommodations in this class, if you have emergency medical information you wish to share with me, or if you need special arrangements in case the building must be evacuated, please inform me immediately: feel free to talk to me privately after class or in my office.