The book references are to do Carmo, *Differential Geometry of Curves and Surfaces*. (The numbers for the assigned problems are the same in both editions of the book.)

**Due:** Thursday, Apr 2, on Gradescope.\(^1\)

**Exercise 1.** Show that at a hyperbolic point of a regular surface, the principal directions bisect the asymptotic directions.

**Exercise 2.** Let \( S \) be a regular surface, \( p \in S \).

1. Show that the sum of the normal curvatures for any pair of orthogonal directions at \( p \) is constant.
2. Show that if the mean curvature at \( p \) is zero, and \( p \) is not a planar point (that is, \( dN_p \neq 0 \)), then \( p \) has two orthogonal asymptotic directions.

**Exercise 3.** A curve \( C \) is called a *line of curvature* of a regular surface \( S \) if each tangent vector of \( C \) is a principal direction of \( S \). Suppose two regular surfaces \( S_1, S_2 \) intersect in a regular curve \( C \), and the angle between the normal vectors of \( S_1 \) and \( S_2 \) is \( \theta(p) \), \( p \in C \). Assume that \( C \) is a line of curvature of \( S_1 \). Show that \( C \) is a line of curvature of \( S_2 \) if and only if \( \theta(p) \) is constant.

**Exercise 4.**
1. Let \( R > 0 \). Suppose \( \alpha : I \to \mathbb{R}^3 \) is a regular parameterized curve in \( \mathbb{R}^3 \) with the property that \( \|\alpha(s)\| \leq R \) and \( \|\alpha(s_0)\| = R \). Show that the curvature of \( \alpha \) at \( s_0 \) satisfies the inequality \( k(s_0) \geq 1/R \).
2. Let \( S \) be a compact (that is, closed and bounded) regular surface. Show that there exists a point \( p \in S \) with positive Gauss curvature.

**Exercise 5.** Let \( I \subset \mathbb{R} \) be an open interval, \( \alpha : I \to \mathbb{R}^3 \) a regular parameterized curve, and \( \beta : I \to \mathbb{R}^3 \) a smooth function with \( \beta \neq 0 \). We define a parameterized surface by
\[
x(u,v) = \alpha(u) + v\beta(u), \quad (u,v) \in I \times \mathbb{R}.
\]
This is called a *ruled surface*, with *rulings* \( \beta \) and *directrix* \( \alpha \). (An example is a cylinder, with \( \alpha \) a circle and \( \beta \) a constant vector.) Show that a regular ruled surface has Gauss curvature \( K \leq 0 \).

**Exercise 6.** Chapter 3–3, Problem 13.

---

\(^{1}\)See the course website, [https://math.mit.edu/~phintz/18.950-S20/](https://math.mit.edu/~phintz/18.950-S20/), for homework policies.