EXERCISES ON ADJUNCTIONS

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Notation. We denote an adjunction by an opposing pair of arrows: \( F : C \rightleftharpoons D : G \). In this notation, \( F \) will always be the left adjoint and \( G \) the right adjoint.

Exercise 1. Let \( F : C \rightleftharpoons D : G \) be an adjunction. Prove that the right adjoint \( G \) is fully faithful if and only if the counit \( \varepsilon : FG \Rightarrow \text{id}_D \) is a natural isomorphism. Notice that in this case the left adjoint \( F \) is essentially surjective on objects. Dually, \( F \) is fully faithful if and only if the unit \( \eta : \text{id}_C \Rightarrow GF \) is a natural isomorphism.