18.06 - Summer 2007, Problem Set 6

July 24, 2007

This problem set is due Wednesday August 1 in class. Make sure to include your **name** in your homework! The numbers of the sections and exercises refer to " Introduction to Linear Algebra, **3rd Edition**, by Gilbert Strang.".

Problems from the book:

- Section 6.4, Problems 2 and 22.
- Section 6.5, Problems 4, 15 and 28.
- Section 6.6, Problems 2, 11 and 20.

Other Problems

C_1 a) Let B be a real symmetric matrix and v a real vector. Suppose that $B^2v = 0$. Show that $Bv = 0$.

C_1 b) Let B be a real symmetric matrix and v a real vector. Suppose that $B^n v = 0$ for some positive integer n. Show that $Bv = 0$. (In this question C_1, you may not assume that B is diagonalisable, since they are part of the steps in proving that an arbitrary symmetric matrix is diagonalisable).

C_2 Suppose you have a 3×3 symmetric positive definite matrix A, with three different eigenvalues $\lambda_1 > \lambda_2 > \lambda_3$. Denote by v_1 the eigenvector associated to λ_1. Show that for 'almost' any vector $v \neq 0$, $\lim_{n\to\infty} A^n v$ is in the same direction as v_1.