Homework List (updated February 28, 2018)
Our homework basically follows Kac 2016

Homework 1, due Wed Feb 14:
- Sect. 2 (pg 25): 1–4, 7–10
- Bonus: Given a set $S$, the group $(\text{Aut}(S),\circ)$ is abelian if and only if $|S| \leq 2$.
- Sect. 4 (pg 45): 2–8
- Sect. 5 (pg 55): 22, 24, 31, 33, 51
- Sect. 6 (pg 66): 4, 17, 19, 21, 28, 32–34
- Prove the following: Suppose $G$ is a cyclic group. Then either $|G| = \infty$ and $G \cong \mathbb{Z}$, or $|G| = n < \infty$ and $G \cong \mathbb{Z}/n\mathbb{Z}$.

Homework 2, due Wed Feb 21:
- Sect. 7 (pg 72): 1, 3, 6
- Sect. 8 (pg 83): 2, 8, 10, 17, 35 only d, g, h, 44, 45, 47
- Sect. 9 (pg 94): 1, 7, 10, 11, 13 only a, b, c, 14, 15, 23, 27 only a, c
- Bonus: Show that the elementary braids $s_1$ and $s_2$ in $B_3$ satisfy $s_1s_2s_1 = s_2s_1s_2$. Show that the analogous transpositions $\tau_1$ and $\tau_2$ in $S_3$ also satisfy $\tau_1\tau_2\tau_1 = \tau_2\tau_1\tau_2$.

Homework 3, due Wed Feb 28:
- Sect. 10 (pg 101): 4, 16, 30–32, 39, 45, 46, 47
- Sect. 13 (pg 133): 1–10, 14, 22, 29, 47, 52
- Bonus: Let $Q_8$ be the quaternion group (wikiwand.com/en/Quaternion_group). This is an order 8 non-abelian group. Show that every subgroup in $Q_8$ is normal. Conclude that there exists no group isomorphism between $D_4$ and $Q_8$.
- Bonus: Show that $A_4$ contains no subgroup of order 6.

Homework 4, due Wed Mar 7:
- Sect. 14 (pg 142): 3, 7, 16, 17–20, 21, 30, 34, 40, 26
- Sect. 15 (pg 151): 13, 34, 37, 38
- Sect. 11 (pg 110): 1, 8, 14, 15
- Find all normal subgroups of $D_4$.
- Provide a group isomorphism between $S^1$ and $S^1/\mu_N$ for arbitrary positive $N$. Here $S^1$ is the subgroup of modulus 1 elements in $\mathbb{C}^\times$, and $\mu_N$ is the normal subgroup of $N$-th roots of 1 in $S^1$ (or in $\mathbb{C}^\times$ if you like).

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