

# Course 18.312: Algebraic Combinatorics

## Problem Set # 5

Due Wednesday March 11th, 2009

You may discuss the homework with other students in the class, but please write the names of your collaborators at the top of your assignment. Please be advised that you should not just obtain the solution from another source. Please explain your reasoning to receive full credit, even for computational questions.

- 1) (10 points) Draw all Standard Young Tableaux of shape  $[2, 2, 1, 1]$ .
- 2) (10 points) Deduce and prove a formula for  $f_{[k, 1^{n-k}]}$ , the number of Standard Young Tableaux of shape  $\lambda = [k, 1^{n-k}]$ .

**Hint:** You may use the hook-length formula, but your ultimate formula should be simpler. For example, the cases of  $k = 0$  and  $1$  have simple formulas  $f_{[n]} = 1$  and  $f_{[n-1, 1]} = n - 1$ .

- 3) (10 points) Using the hook-length formula, prove that the number of Standard Young Tableaux of shape  $\lambda = [n, n]$  is given by

$$C_n = \frac{1}{n+1} \binom{2n}{n}.$$

The  $C_n$ 's are known as the **Catalan** numbers, and provide the answer to a number of questions in combinatorics.

- 4) (10 points) What is the image of  $RSK(\pi)$  where  $\pi(1) = 4, \pi(2) = 3, \pi(3) = 2, \pi(4) = 6, \pi(5) = 5, \pi(6) = 1$ ?

i.e.

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 2 & 6 & 5 & 1 \end{pmatrix}.$$

(10 points) What is the image of

$$RSK^{-1} \left( \begin{array}{ccc|ccc} 1 & 3 & 6 & 1 & 4 & 5 \\ 2 & 5 & 7, & 2 & 6 & 7 \\ 4 & & & 3 & & \end{array} \right)?$$