

Course 18.312: Algebraic Combinatorics

Homework # 10

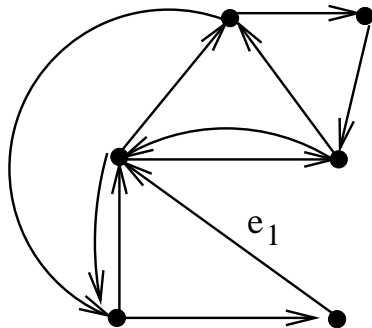
Due Friday May 1, 2009

You may discuss the homework with other students in the class, but please write the names of your collaborators at the top of your assignment. Please be advised that you should not just obtain the solution from another source. Please explain your reasoning to receive full credit, even for computational questions.

1) Let D be the digraph pictured in the figure depicted below.

(10 points) How many Eulerian tours are there in D starting with edge e_1 ?

(**Note:** You may use a computer algebra package for this problem, but make sure to describe your calculations.)



2) Let $G = P_6$, the path graph on vertices $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ with v_i adjacent to v_{i-1} and v_{i+1} for $2 \leq i \leq 5$. Consider the initial configuration C which contains N chips on vertex v_1 and zero chips on all other vertices.

(5 points) For what values of N does the firing process go on infinitely, and for what values of N does the firing process terminate?

(10 points) For the values of N for which the process terminates, describe the possible resulting configurations.

- 3) Let $G = C_n$ be a cycle graph on n vertices and let v_0 be one of G 's vertices.
- (5 points) How many critical configurations does C_n have, letting v_0 be the sink vertex?
- (10 points) Describe the critical configurations of C_n . (**Hint:** To get started, try writing down the critical configurations for small n .)
- (10 points) What is the critical group $K(C_n, v_0)$, written as a product of cyclic groups?