

**FIRST PRACTICE MIDTERM
MATH 18.703, MIT, SPRING 13**

You have 80 minutes. This test is closed book, closed notes, no calculators.

There are 7 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.* Points will be awarded on the basis of neatness, the use of complete sentences and the correct presentation of a logical argument.

Name:_____

Signature:_____

Student ID #:_____

Problem	Points	Score
1	15	
2	15	
3	20	
4	10	
5	10	
6	15	
7	10	
Presentation	5	
Total	100	

1. (15pts) (i) Give the definition of a normal subgroup.

(ii) Give the definition of a group homomorphism.

(iii) Give the definition of A_n , the alternating group.

2. (15pts) (i) Exhibit a proper normal subgroup V of A_4 . To which group is V isomorphic to?

(ii) Give the left cosets of V inside A_4 .

(iii) To which group is A_4/V isomorphic to?

3. (20pts) Let G be a group and let H be a subgroup. Prove that the following are equivalent.

- (1) H is normal in G .
- (2) For every $g \in G$, $gHg^{-1} = H$.
- (3) For every $a \in G$, $aH = Ha$.
- (4) The set of left cosets is equal to the set of right cosets.

4. (10pts) Let G be a group and let N be a normal subgroup. Prove that G/N is abelian iff N contains the commutator of every pair of elements of G .

5. (10pts) Let H and K be two normal subgroups of a group G , whose intersection is the trivial subgroup. Prove that every element of H commutes with every element of K .

6. (15pts) (i) State the Sylow Theorems.

(ii) Prove that if G is a group of order pq , where p and q are distinct primes, then G is not simple.

7. (10pts) Let G be a group and let N be a normal subgroup. Show that there is a natural bijection between the set of subgroups H of G which contain N and the subgroups of the quotient group G/N . Show that this bijection preserves normality, so that normal subgroups of G which contain N correspond to normal subgroups of G/N .