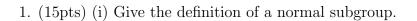
FIRST PRACTICE MIDTERM MATH 18.703, MIT, SPRING 13

You have 80 minutes. This test is closed book, closed notes, no calculators.

There are 7 problems, and the total number of points is 100. Show all your work. Please make your work as clear and easy to follow as possible. Points will be awarded on the basis of neatness, the use of complete sentences and the correct presentation of a logical argument.

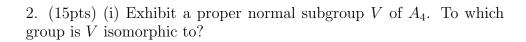
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Problem	Points	Score
1	15	
2	15	
3	20	
4	10	
5	10	
6	15	
7	10	
Presentation	5	
Total	100	



(ii) Give the definition of a group homomorphism.

(iii) Give the definition of A_n , the alternating group.



(ii) Give the left cosets of V inside A_4 .

(iii) To which group is A_4/V isomorphic to?

- 3. (20pts) Let G be a group and let H be a subgroup. Prove that the following are equivalent.
 - (1) H is normal in G.
 - (2) For every $g \in G$, $gHg^{-1} = H$. (3) For every $a \in G$, aH = Ha.

 - (4) The set of left cosets is equal to the set of right cosets.

4. (10pts) Let G be a group and let N be a normal subgroup. Prove that G/N is abelian iff N contains the commutator of every pair of elements of G.

5. (10pts) Let H and K be two normal subgroups of a group G, whose intersection is the trivial subgroup. Prove that every element of H commutes with every element of K.

6. (15pts) (i) State the Sylow Theorems.

(ii) Prove that if G is a group of order pq, where p and q are distinct primes, then G is not simple.

7. (10pts) Let G be a group and let N be a normal subgroup. Show that there is a natural bijection between the set of subgroups H of G which contain N and the subgroups of the quotient group G/N. Show that this bijection preserves normality, so that normal subgroups of G which contain N correspond to normal subgroups of G/N.